The Wigner–Smith Operator: A Key Tool for Applications in Wave Physics

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Quantum Analogues
Luxembourg

TU WIEN

Quantum mechanics

$$H\psi = i\hbar\frac{\partial}{\partial t}\psi$$

$$[x, p] = i\hbar$$

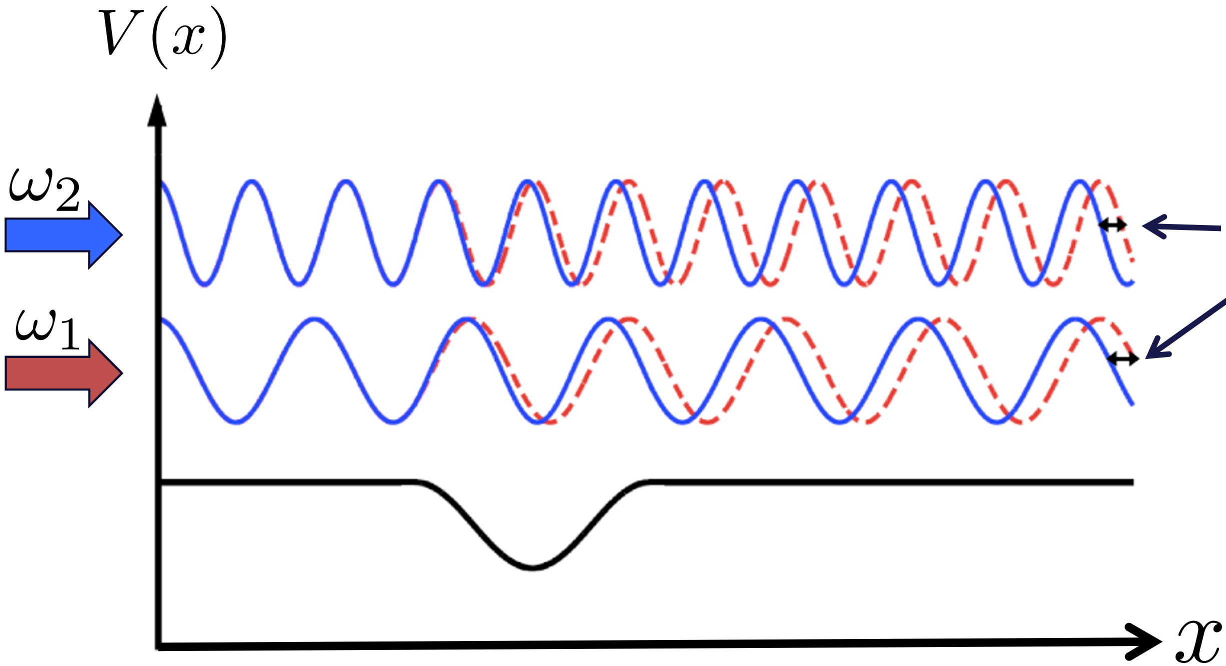
no proper Hermitian
operator for time

Time in scattering

Wigner, Phys. Rev. 98, 145 (1955)
Smith, Phys. Rev. 118, 349 (1960)

$$t(\omega) = e^{i\phi(\omega)}$$



Scattering phase $\phi(\omega)$

time delay $\tau_\phi = \frac{\mathrm{d}\phi(\omega)}{\mathrm{d}\omega} \longrightarrow \tau_\phi = -it^{-1}(\omega)\frac{\mathrm{d}t(\omega)}{\mathrm{d}\omega}$

Multi-channel scattering problems

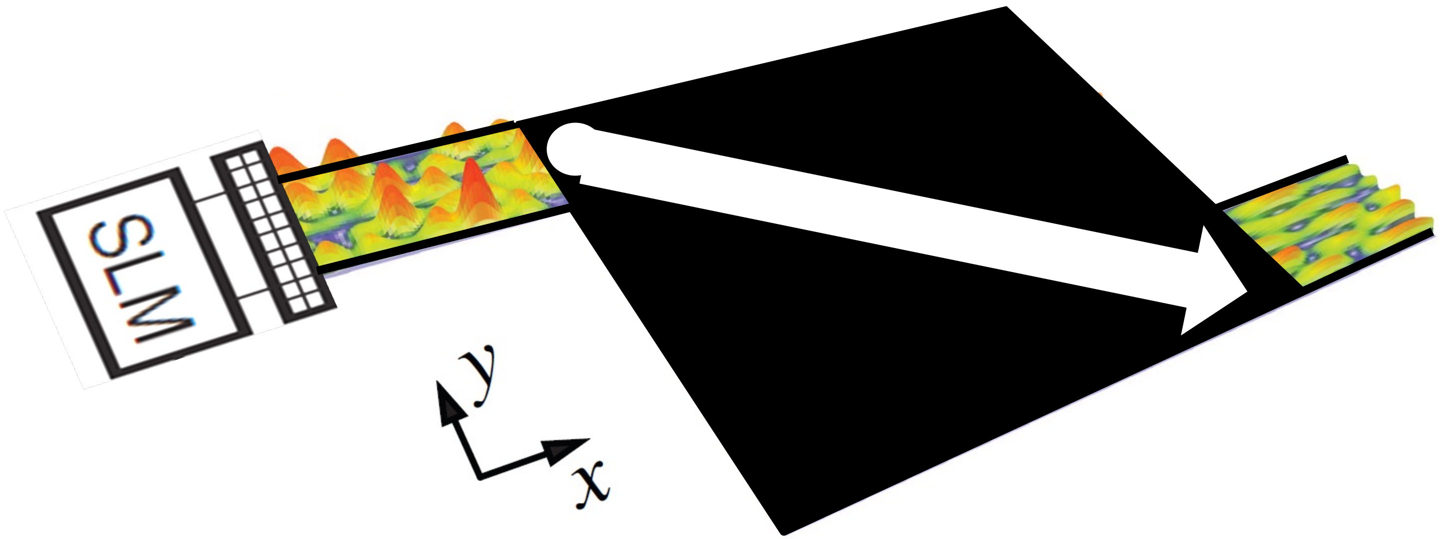
Wigner-Smith time-delay matrix

$$Q = -i\, S^{\dagger} \frac{\partial S}{\partial \omega}$$

Hermitian operator for unitary S-matrix

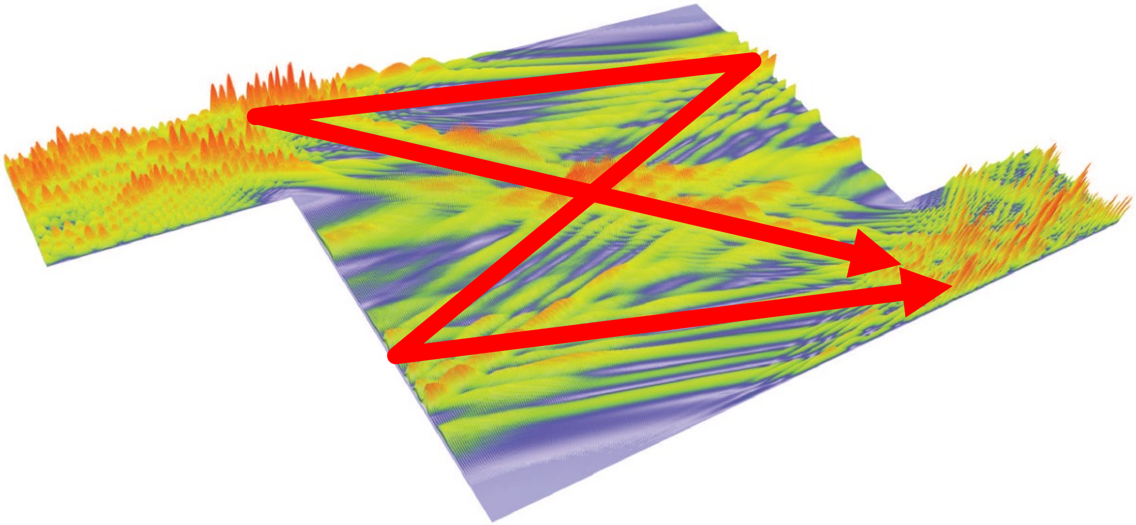
- real eigenvalues (proper delay times)
- orthogonal eigenstates
- no information on scattering potential required
- connection to density of states

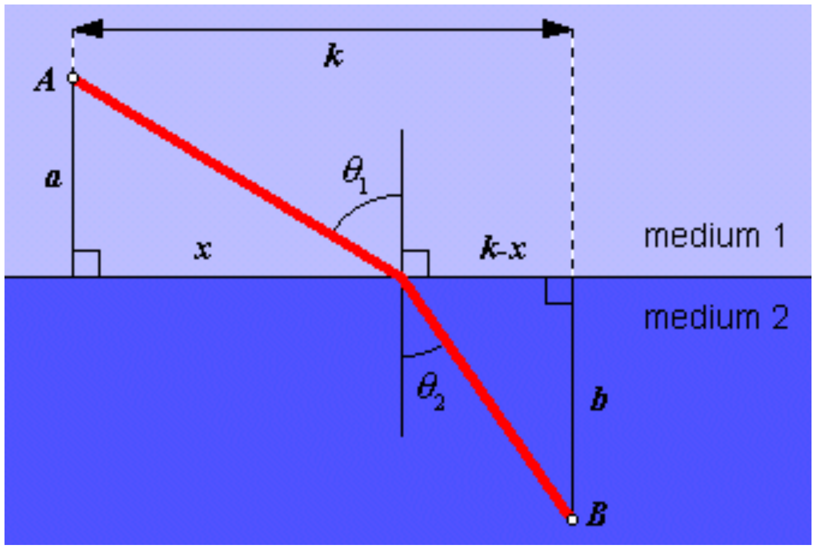
Wave generation

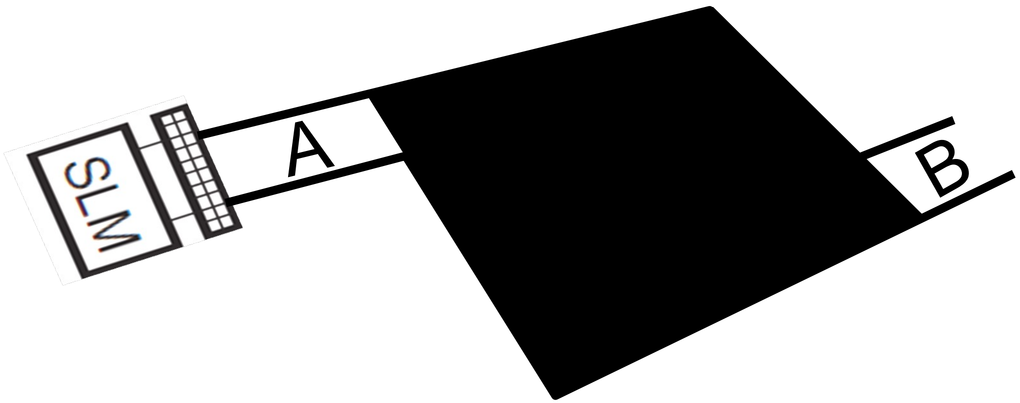


Popoff, Lerosey, Carminati, Fink, Boccara, and Gigan, PRL 104, 100601 (10).

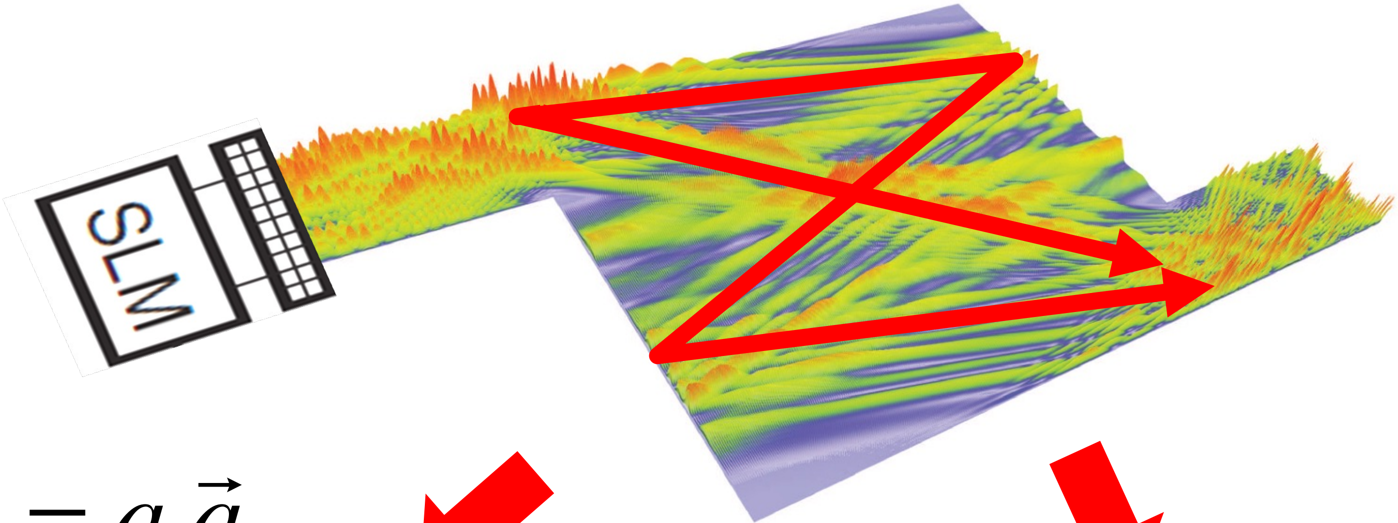
Particle-like states



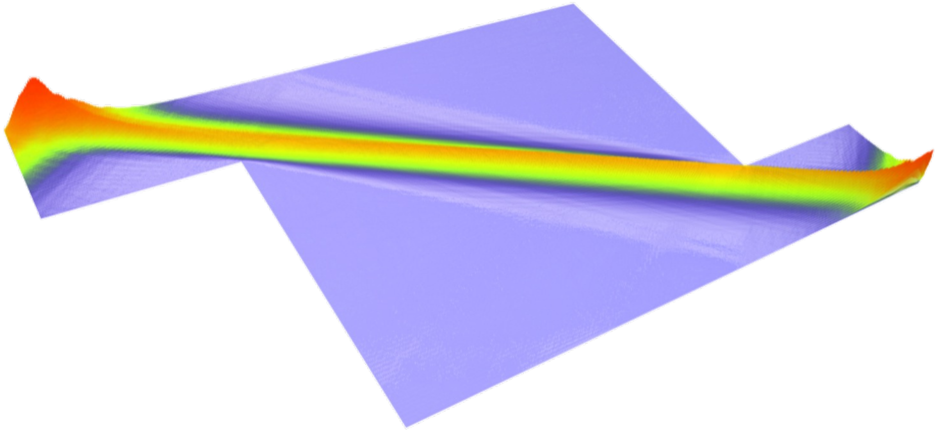


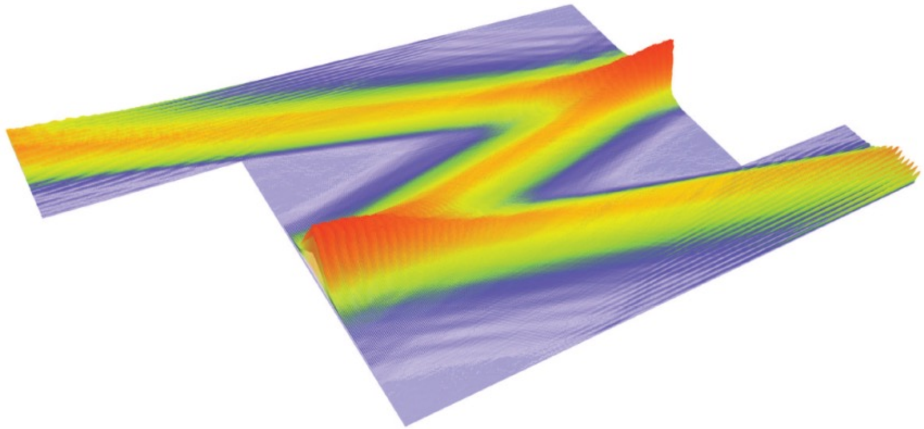


Particle-like states



$$Q\,\vec{q}_i = q\,\vec{q}_i$$



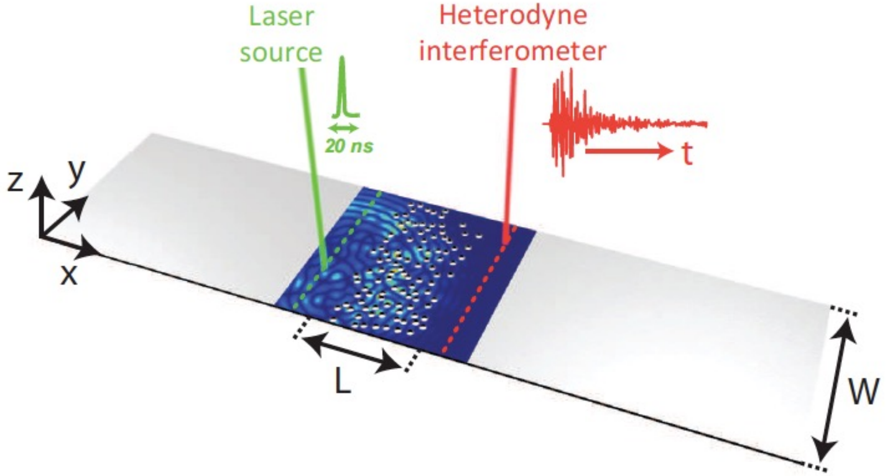


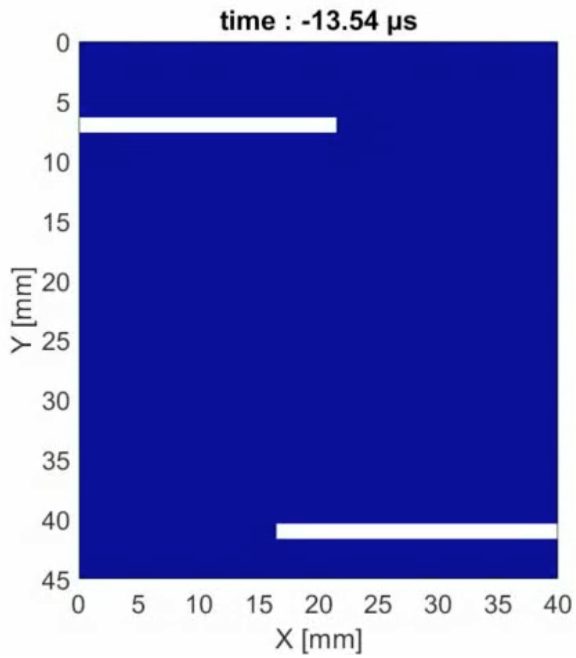
Rotter, Ambichl, Libisch, PRL 106, 120602 (2011).

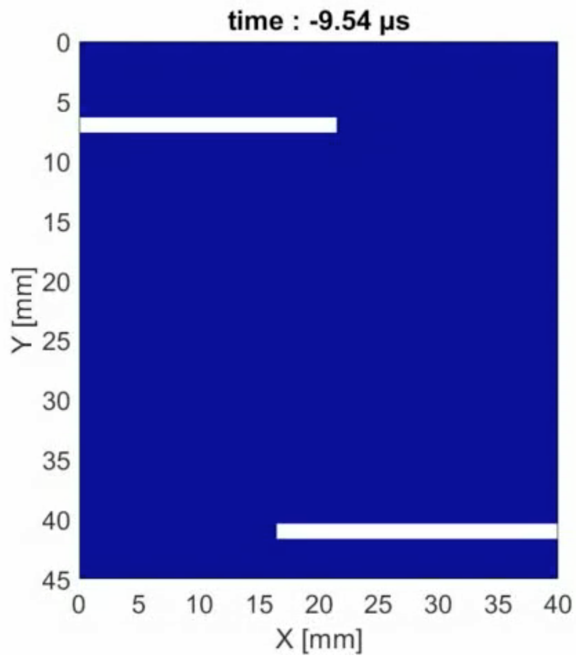
Physical Review Focus

Experiment in acoustics

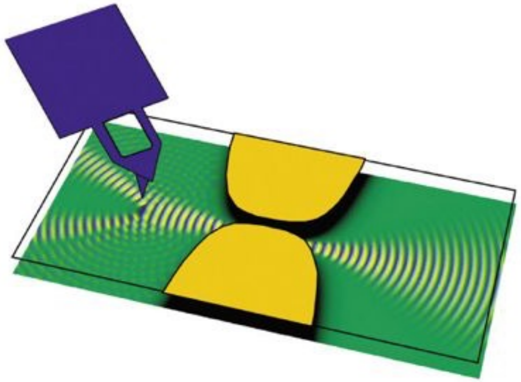
Gérardin, Laurent, Ambichl, Prada, Rotter, and Aubry, PRB 94, 014209 (2016).

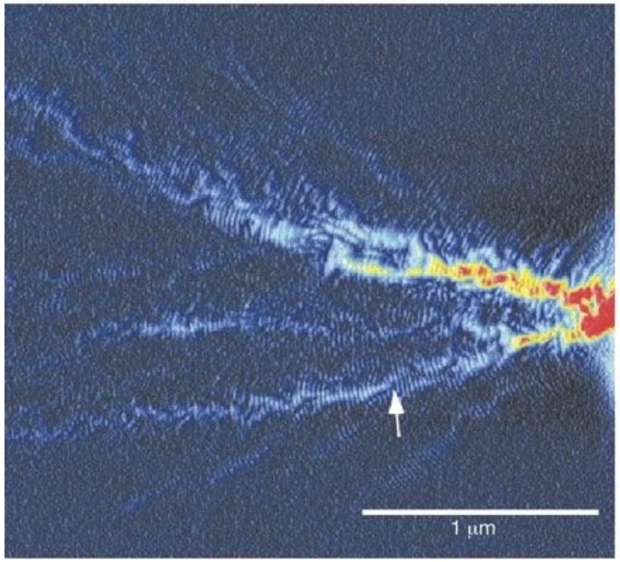






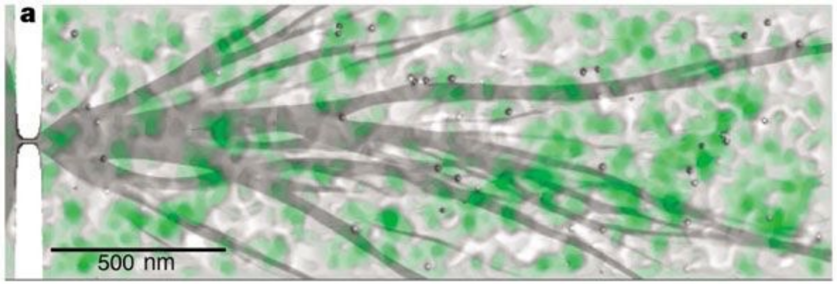
Branched flow

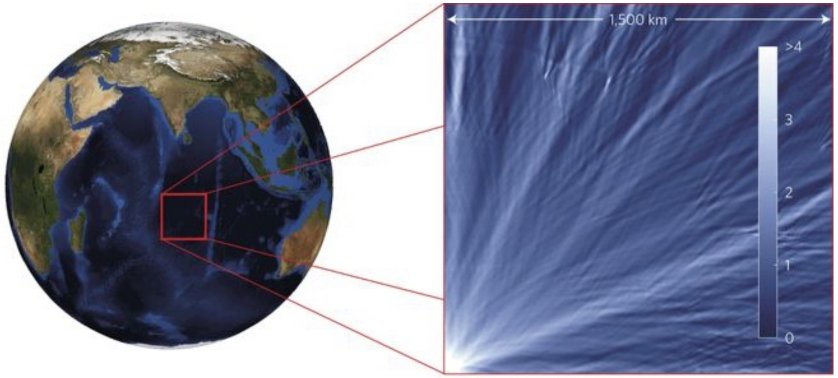


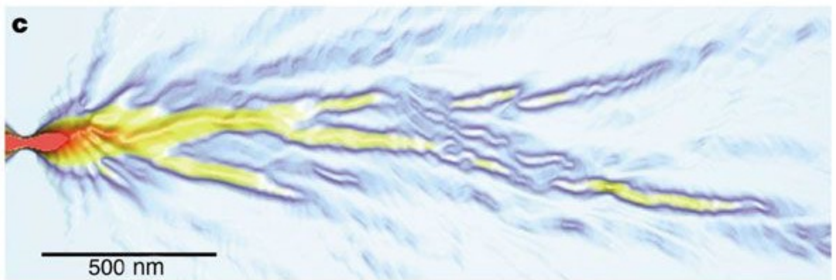


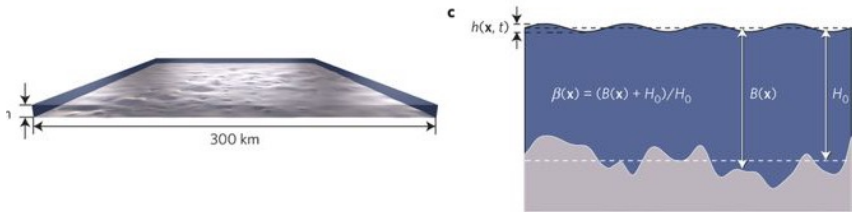
Topinka et al.,
Nature 410, 183 (2001).

Degueldre, Metzger,
Geisel & Fleischmann
Nature Phys. 12, 259 (2016).



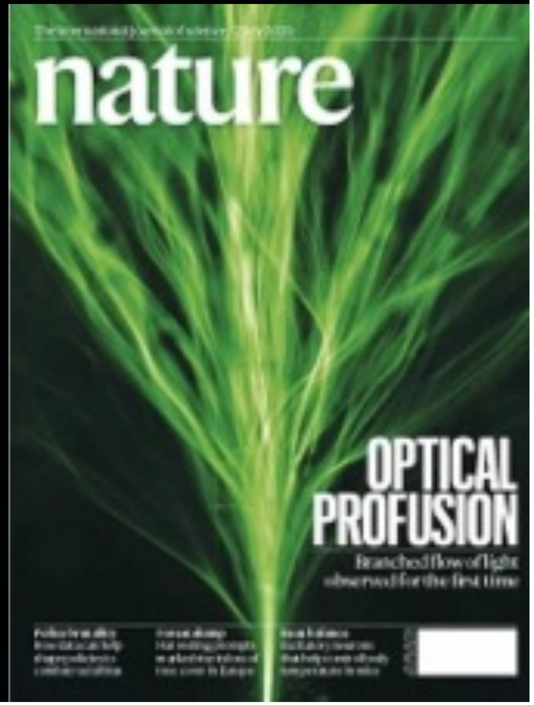




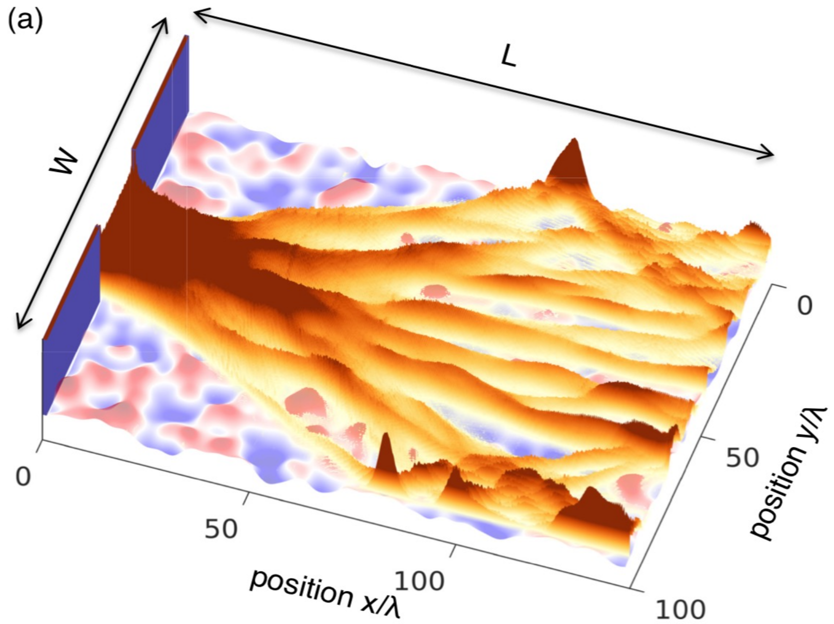


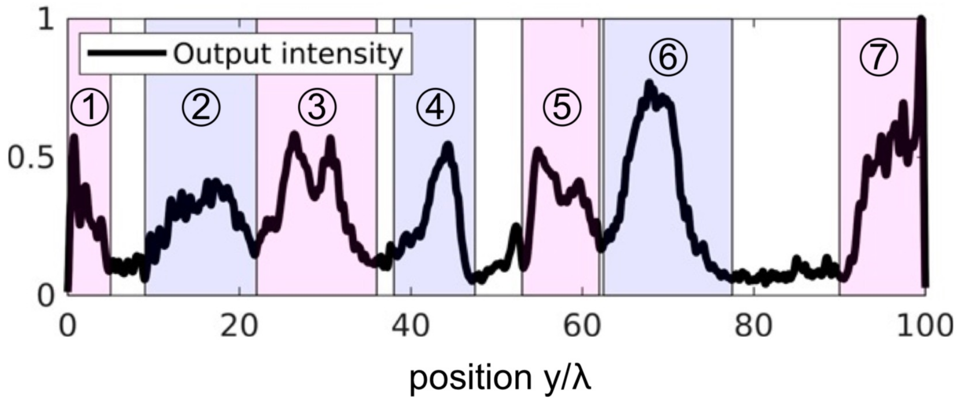
Branched flow in optics

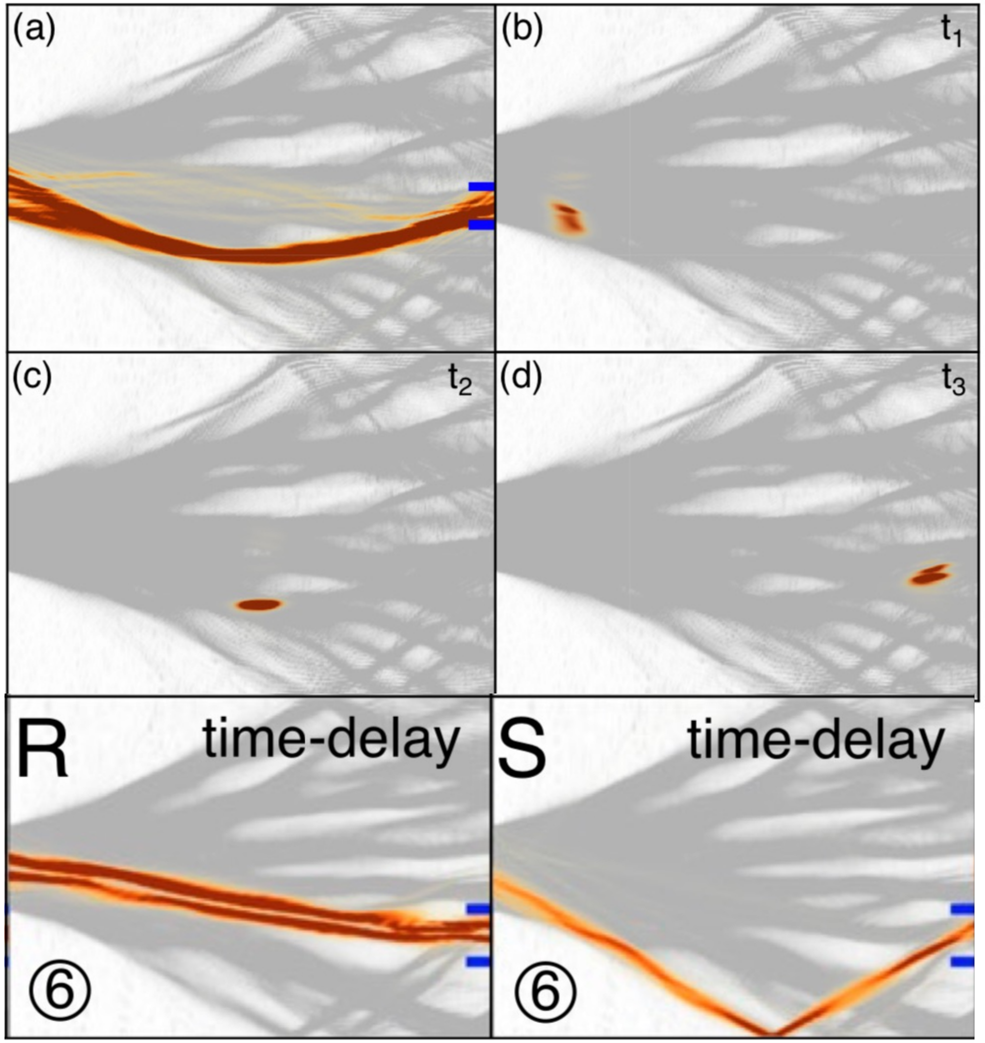
Patsyk, Sivan, Segev & Bandres,
Nature 583, 60 (2020).



Branched flow in optics

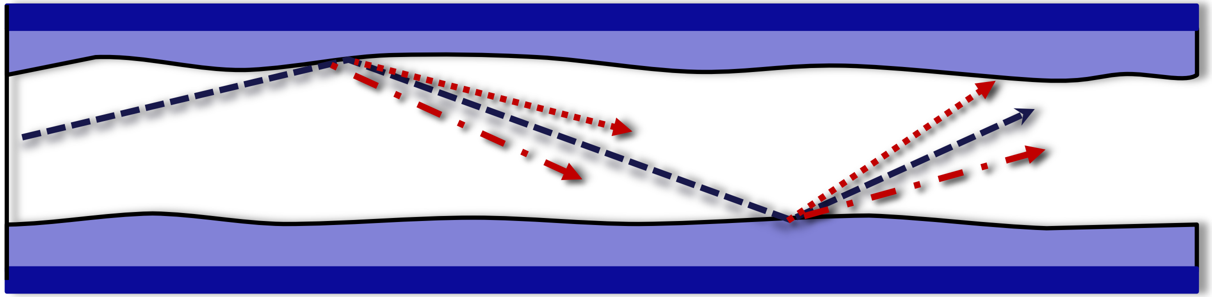




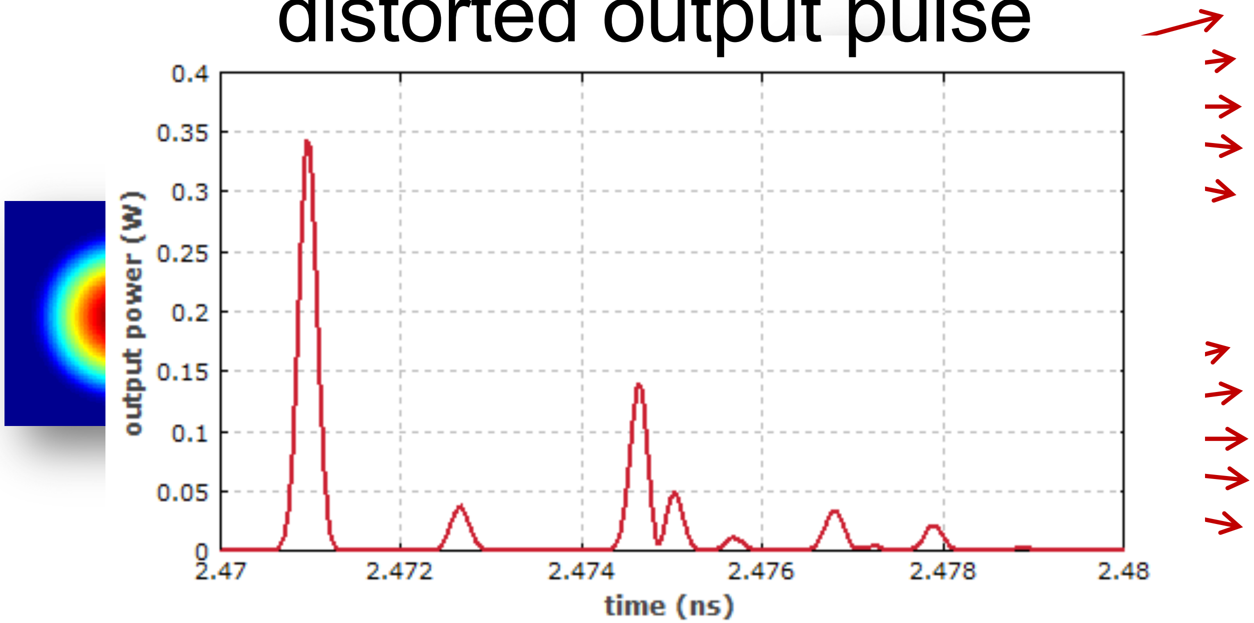


Brandstötter, Girschik, Ambichl & Rotter
PNAS 116, 13260 (2019).

Mode mixing in a multimode fiber

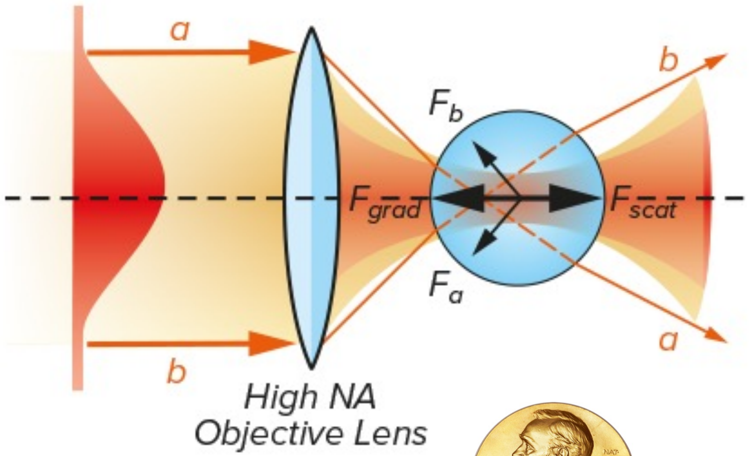


distorted output pulse



Fan & Kahn, Opt. Lett. 30,135 (2005); Carpenter, Eggleton, Schröder, Nat. Phot. 9, 751 (2015).
Xiong, Ambichl, Bromberg, Redding, Rotter, and Cao
PRL 117, 053901 (2016); Optics Express 25, 2709 (2017); PRX 7, 041053 (2017).

Optical micromanipulation



Ashkin, Dziedzic,
Bjorkholm, Chu,
Opt. Lett. 11, 5, 288 (1986).

- Challenges:
  - Complex shaped target
  - Disordered environment
  - Optimality
- Solution:
  Wigner-Smith operator

Generalized Wigner-Smith operator

$$Q = -iS^{-1}(\omega)\frac{\mathrm{d}S(\omega)}{\mathrm{d}\omega} \longrightarrow$$ Well-defined delay times q_i

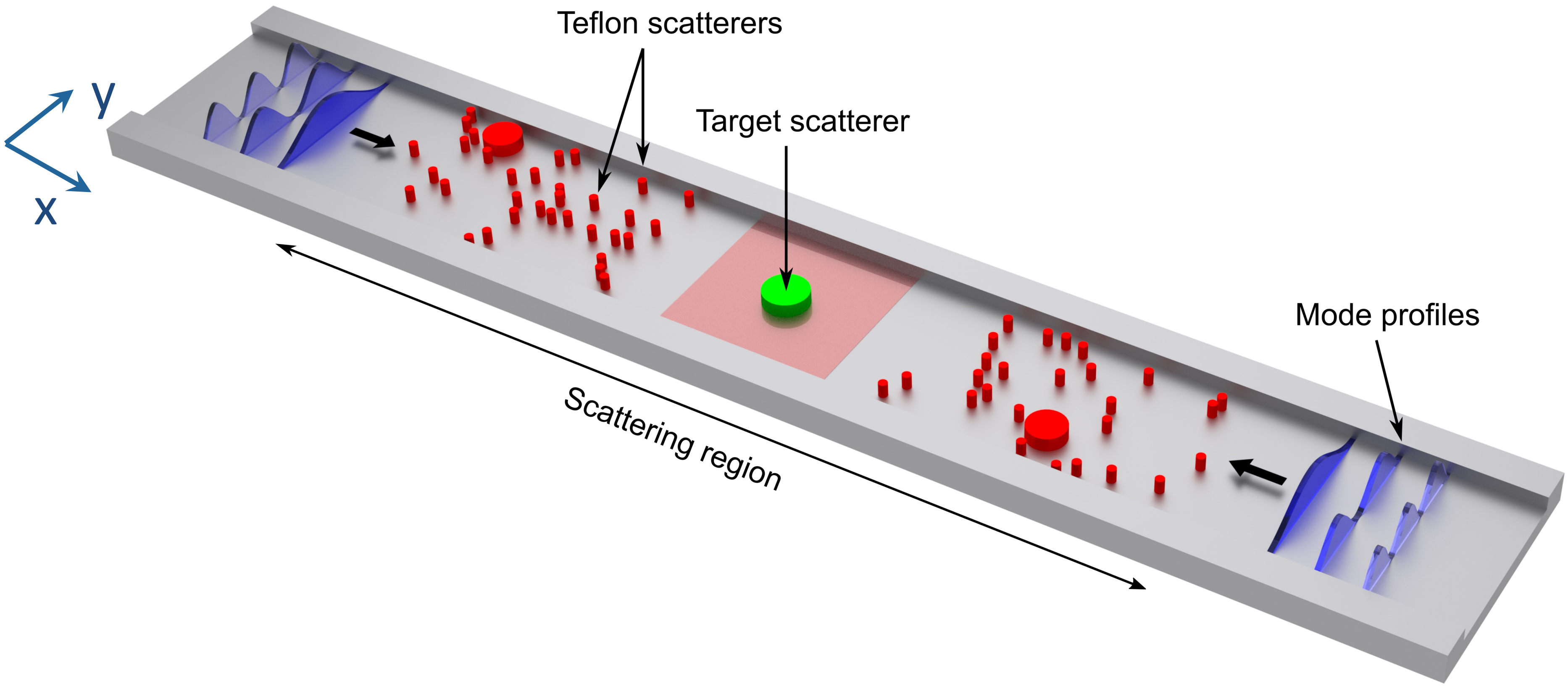
$$\Downarrow$$

$$Q = -iS^{-1}(\alpha)\frac{\mathrm{d}S(\alpha)}{\mathrm{d}\alpha} \longrightarrow$$ Well- defined "$\mathrm{conj}(\alpha)$"

e.g. α = position $\longrightarrow \mathrm{conj}(\alpha)$ = momentum

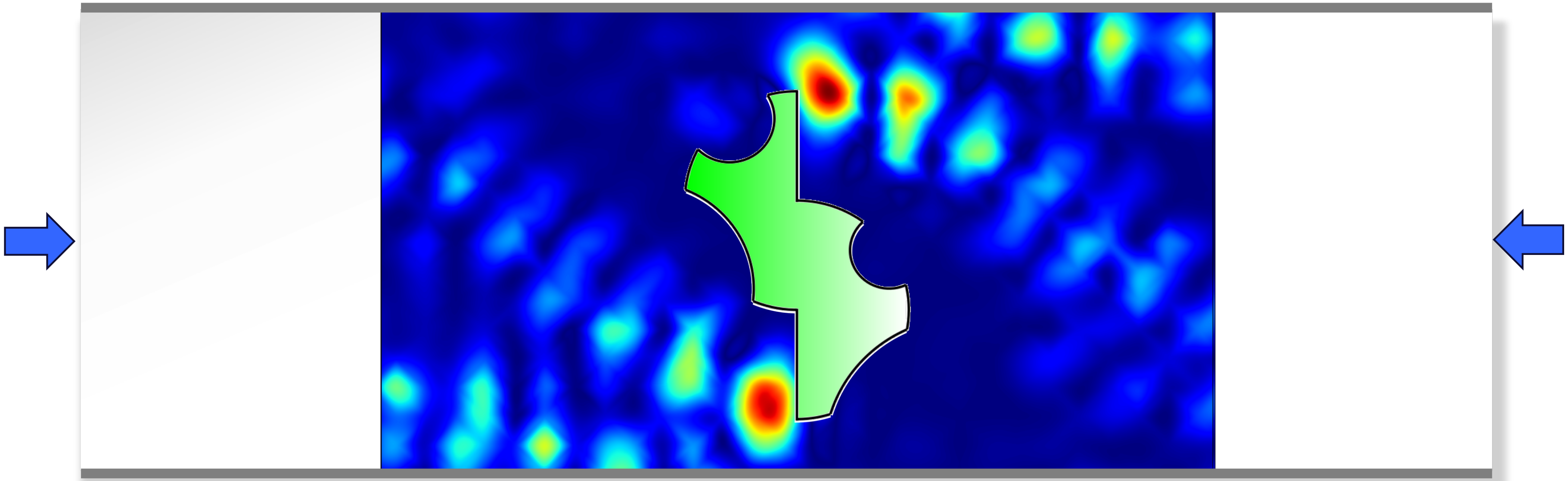
Ambichl, Brandstötter, Böhm, Kühmayer, Kuhl & Rotter
PRL 119, 033903 (2017).

Microwave setup



Optimal angular momentum transfer

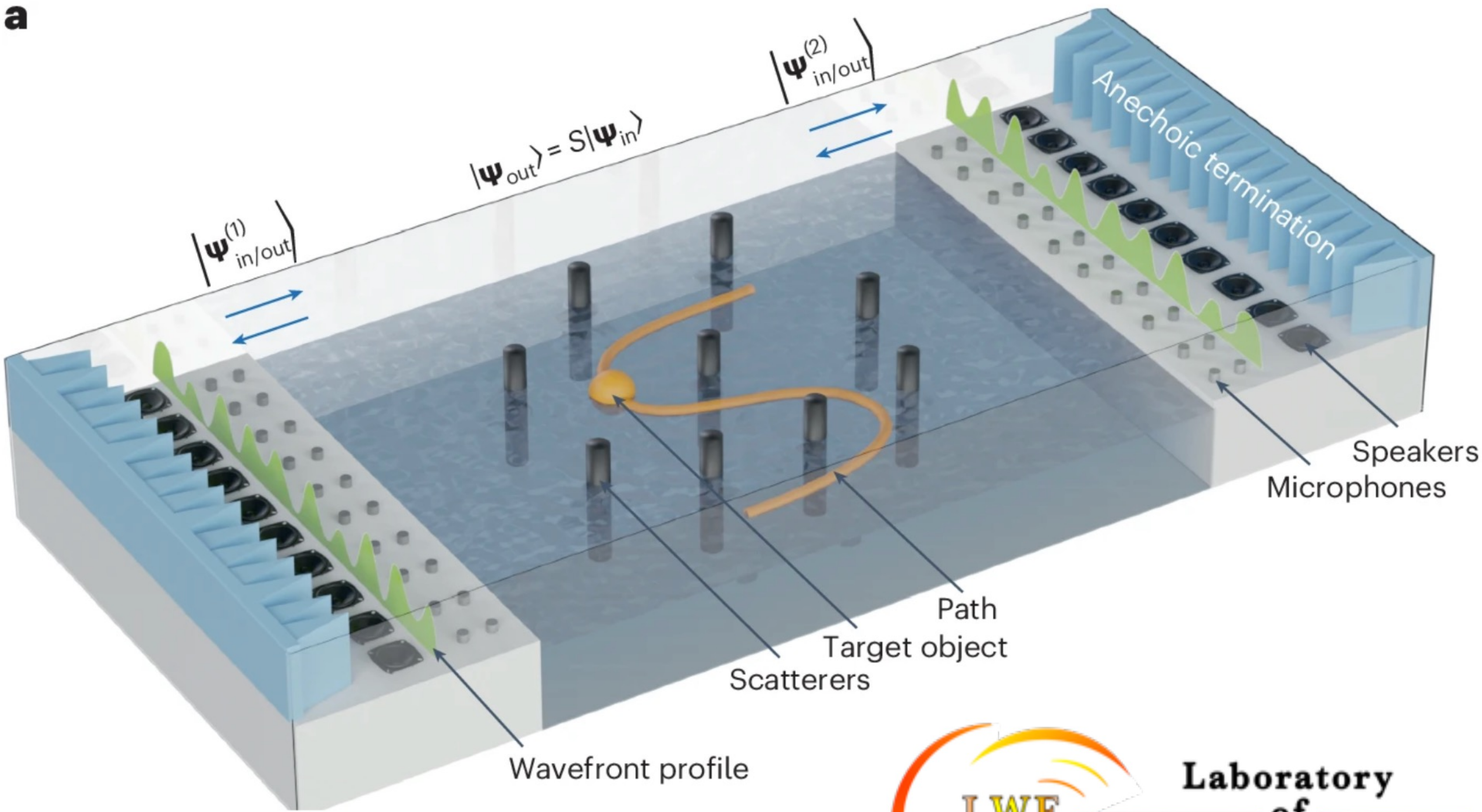
- α = angle $\longrightarrow$ Eigenstate with largest eigenvalue



$$Q = -iS^{-1}(\alpha)\frac{S(\alpha)}{d\alpha}$$

Horodynski, Kühmayer, Brandstötter, Pichler, Fyodorov, Kuhl & Rotter,
Nature Photonics 14, 149 (2020).

Particle manipulation with sound

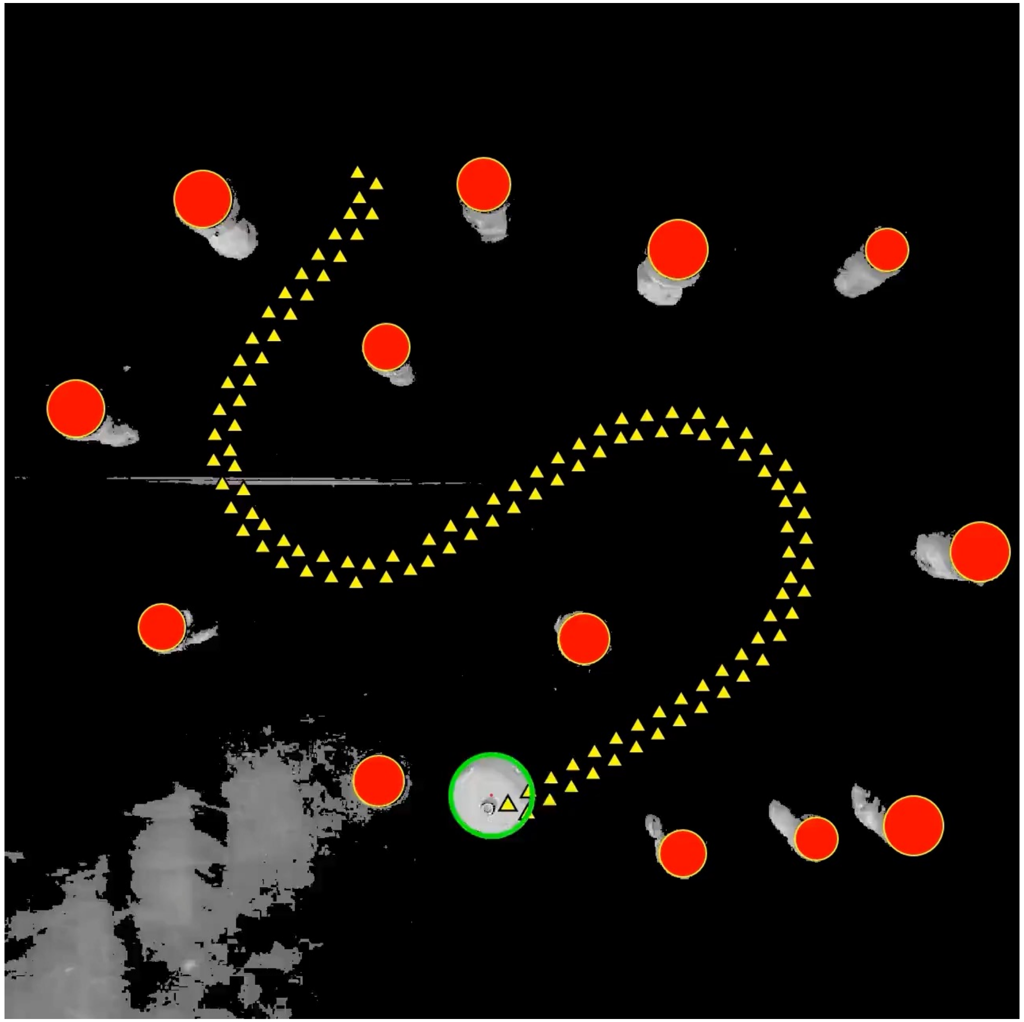


LWE
Laboratory of Wave Engineering

A “loud” experiment

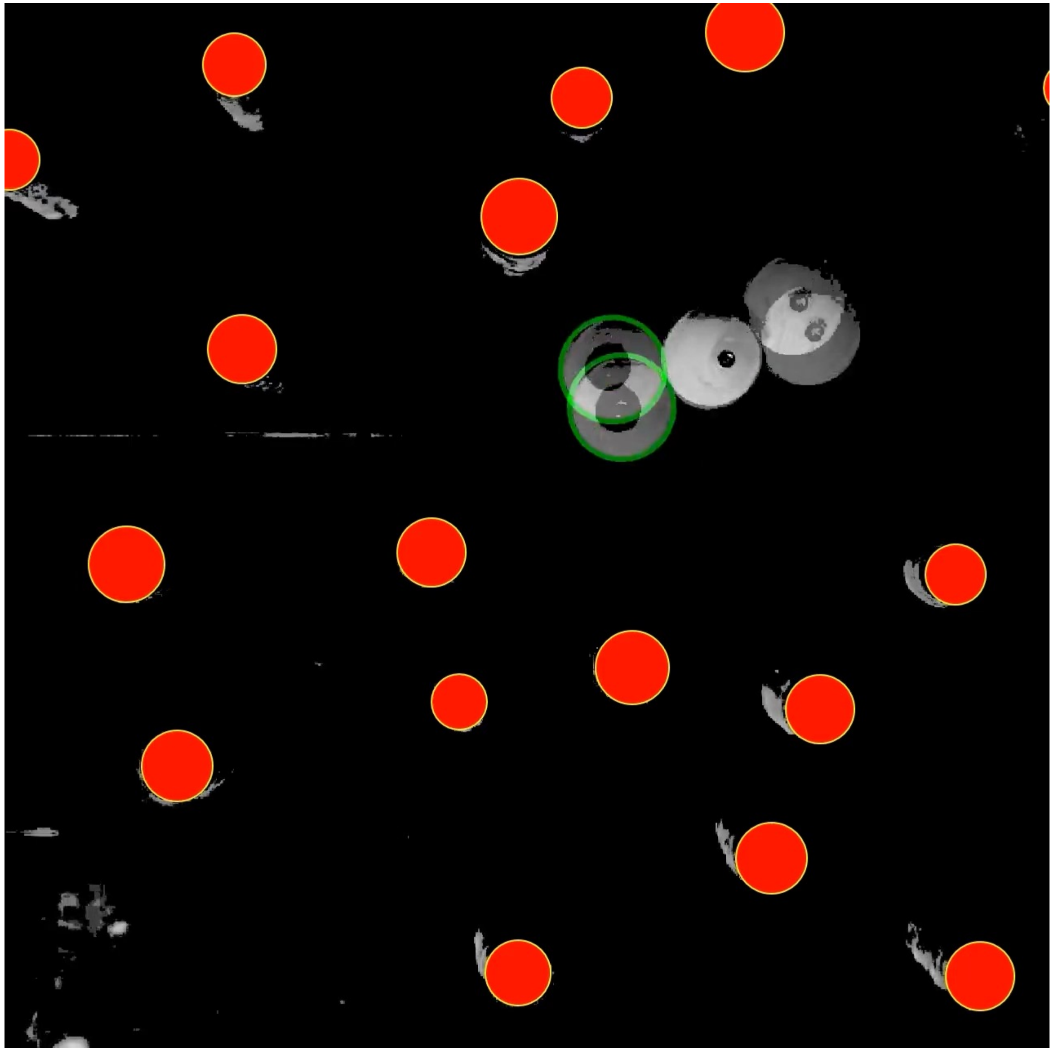
Orazbayev, Malléjac, Bachelard, Rotter & Fleury
Nature Physics 20, 1441 (2024)

Moving a ping-pong ball



Orazbayev, Malléjac, Bachelard, Rotter & Fleury
Nature Physics 20, 1441 (2024)

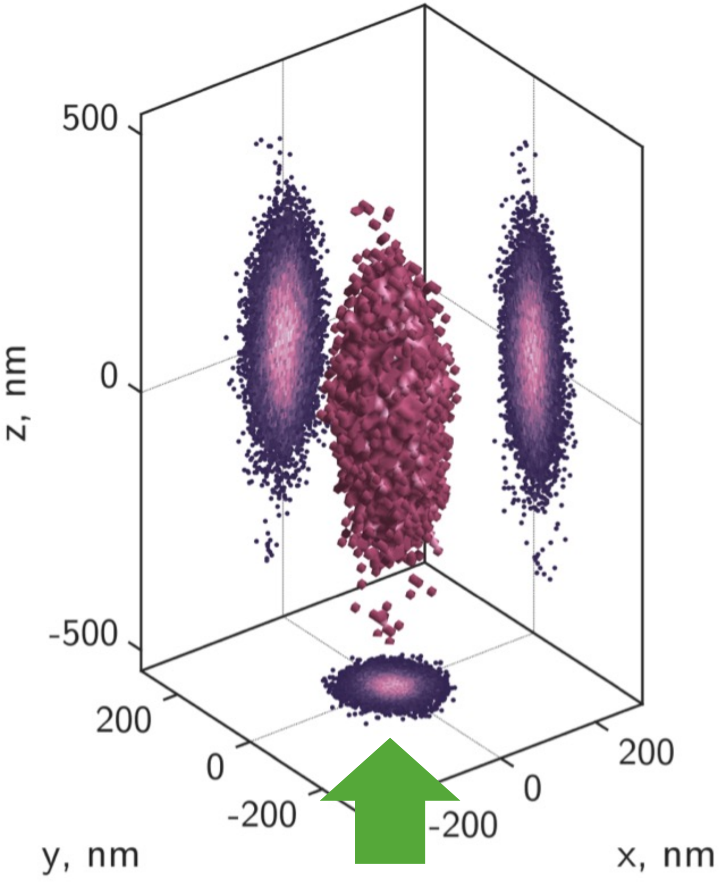
Rotating an object

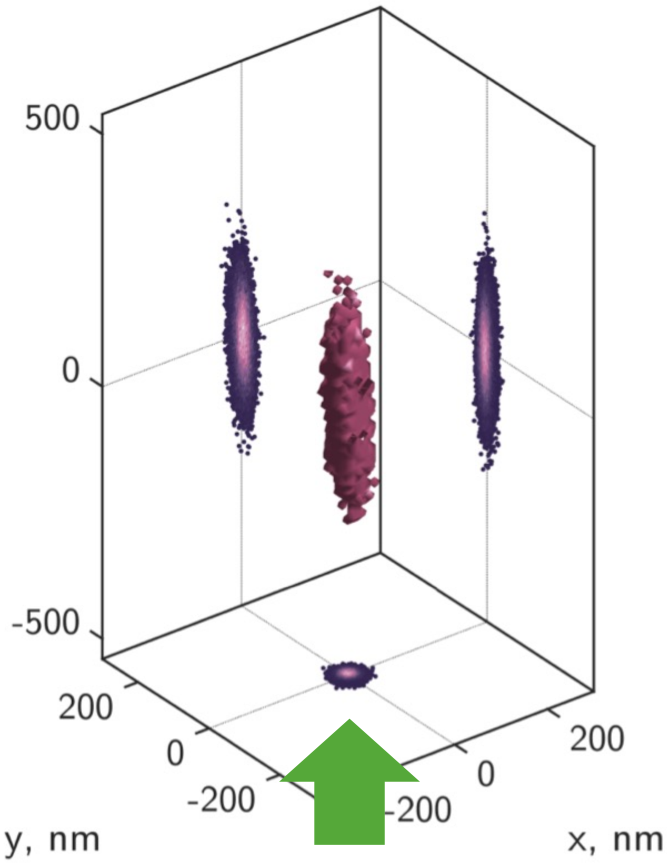


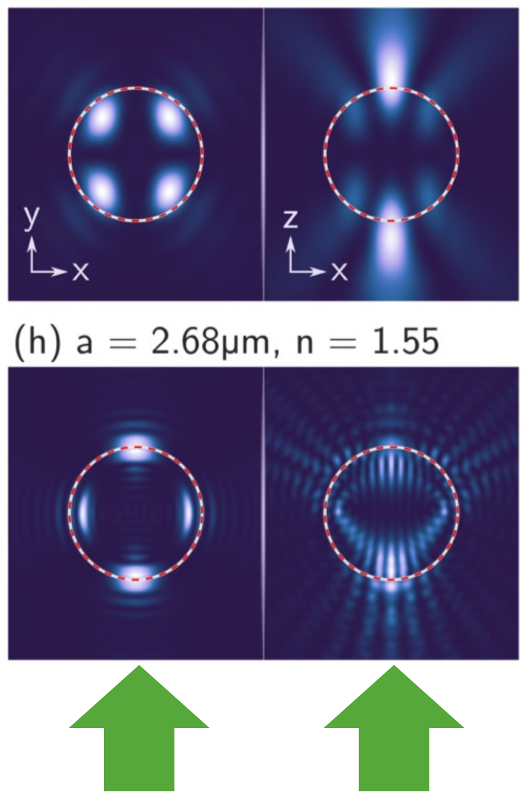
Orazbayev, Malléjac, Bachelard, Rotter & Fleury
Nature Physics 20, 1441 (2024)

Traps with optimized trapping stiffness

$$Q_x = -i\, S^{-1}(x)\,\frac{dS(x)}{dx} \longrightarrow \frac{dQ_x}{dx}$$ Trapping stiffness







Būtaitė, Sharp, Horodynski, Gibson, Padgett, Rotter, Taylor & Phillips, Science Advances 10, eadi7792 (2024)

The Quantum Wigner Smith Operator

$$\mathbf{S}\left[\hat{a}\right] = \hat{U}^{\dagger}\left[\hat{a}\right]\hat{U} \qquad \hat{Q}_{\theta} := -\mathrm{i}\hat{U}^{\dagger}\left(\theta\right)\partial_{\theta}\hat{U}\left(\theta\right)$$

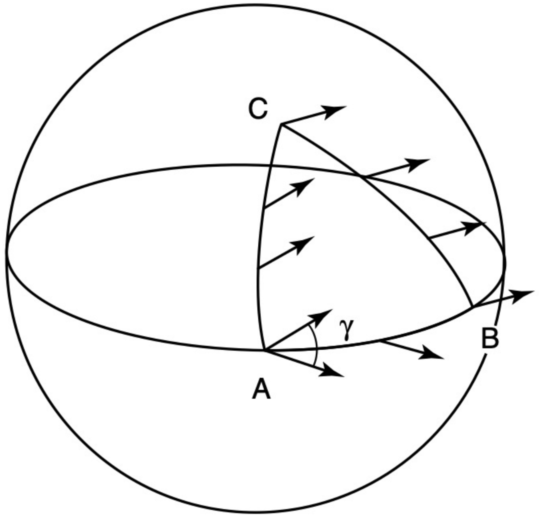
$$\hat{Q}_{\theta} = \left[\hat{a}^{\dagger}\right]^{\top}\mathbf{Q}_{\theta}\left[\hat{a}\right] + \frac{1}{2}\,\mathrm{tr}\left(\mathbf{Q}_{\theta}\right) \qquad \langle\mathbf{0}|\hat{Q}_{\theta}|\mathbf{0}\rangle = \frac{1}{2}\,\mathrm{tr}\left(\mathbf{Q}_{\theta}\right)$$

Hendrik Casimir

Dirk Polder

Rachbauer, Bouchet, Leonhardt & Rotter, J. Opt. Soc. Am. B 41, 2122 (2024).
Lambrecht, Neto & Reynaud, New J. Phys. 8, 243 (2006).
Hörner, Rachbauer, Rotter & Leonhardt, Phys. Rev. B 108, 235430 (2023).

Geometric phase



Bliokh, Kuang, Rotter, arXiv:2506.07144

$$\Delta\boldsymbol{\pi} = \langle\psi_{\text{in}}|\hat{\mathbf{Q}}|\psi_{\text{in}}\rangle \qquad \hat{\mathbf{Q}} = -i\hat{S}^{\dagger}\frac{\partial\hat{S}}{\partial\boldsymbol{\rho}}$$

$$\hat{U}^{\dagger}(\boldsymbol{\rho})\hat{S}(\boldsymbol{\rho})\hat{U}(\boldsymbol{\rho}) = \hat{S}'(\boldsymbol{\rho}) =$$

$$= \text{diag}(s_1(\boldsymbol{\rho}), \ldots, s_n(\boldsymbol{\rho}))$$

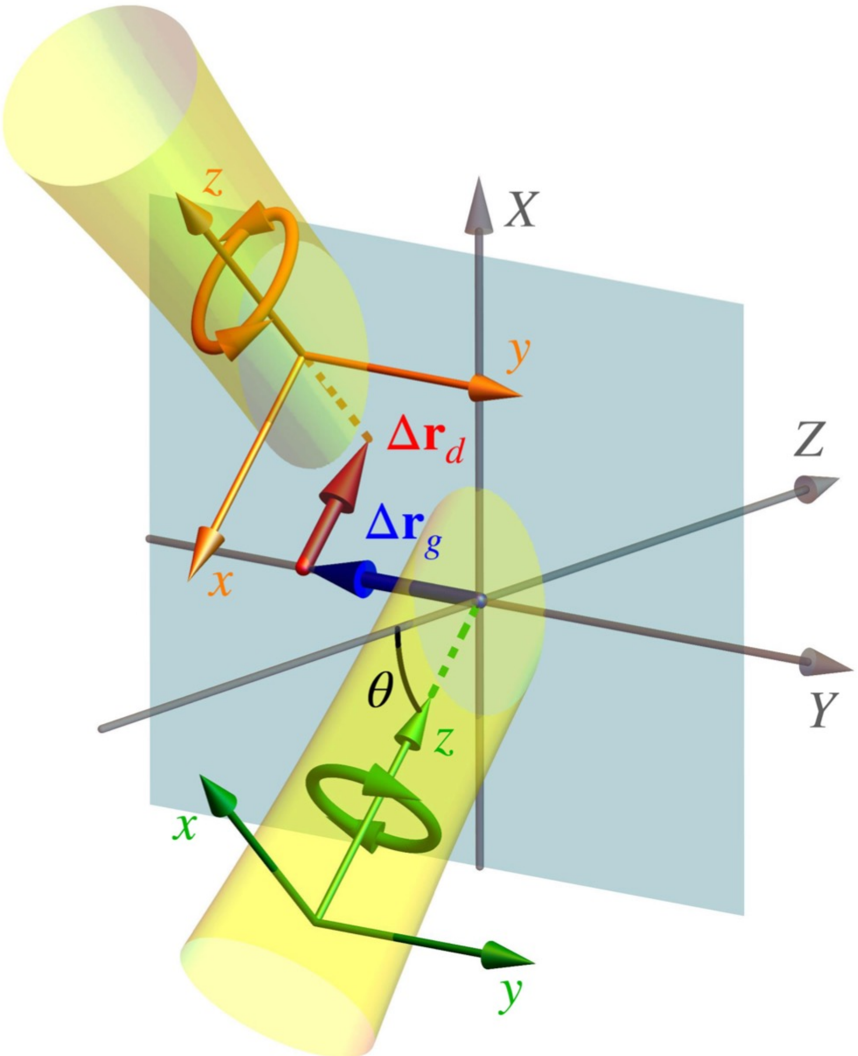
with $s_i(\boldsymbol{\rho}) = e^{i\alpha_i(\boldsymbol{\rho})}$

$$|\psi\rangle = \hat{U}|\psi'\rangle \qquad \hat{\mathbf{Q}}' = -i\hat{S}'^{\dagger}\frac{\partial\hat{S}'}{\partial\boldsymbol{\rho}} = \text{diag}\left(\frac{\partial\alpha_1}{\partial\boldsymbol{\rho}}, \ldots, \frac{\partial\alpha_n}{\partial\boldsymbol{\rho}}\right)$$

$$\Delta\boldsymbol{\pi} = \underbrace{\langle\psi'_{\text{in}}|\hat{\mathbf{Q}}'|\psi'_{\text{in}}\rangle}_{\text{dynamic}} + \underbrace{\langle\psi'_{\text{out}}|\hat{\mathbf{A}}|\psi'_{\text{out}}\rangle - \langle\psi'_{\text{in}}|\hat{\mathbf{A}}|\psi'_{\text{in}}\rangle}_{\text{geometric}}$$

with Berry connection: $\hat{\mathbf{A}} = -i\hat{U}^{\dagger}\dfrac{\partial\hat{U}}{\partial\boldsymbol{\rho}}$

Transverse beam shifts



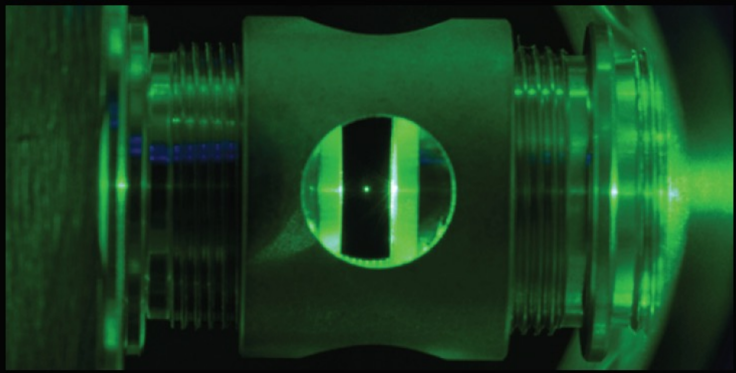
$$\hat{\mathbf{Q}} = i\hat{S}^\dagger \frac{\partial \hat{S}}{\partial \mathbf{k}_\perp}$$

$\Delta \boldsymbol{r}_{\boldsymbol{d}}$: Goos-Hänchen shift is purely dynamical

$\Delta \boldsymbol{r}_{\boldsymbol{g}}$: Imbert-Fedorov shift is purely geometric

Bliokh, Kuang, Rotter, arXiv:2506.07144

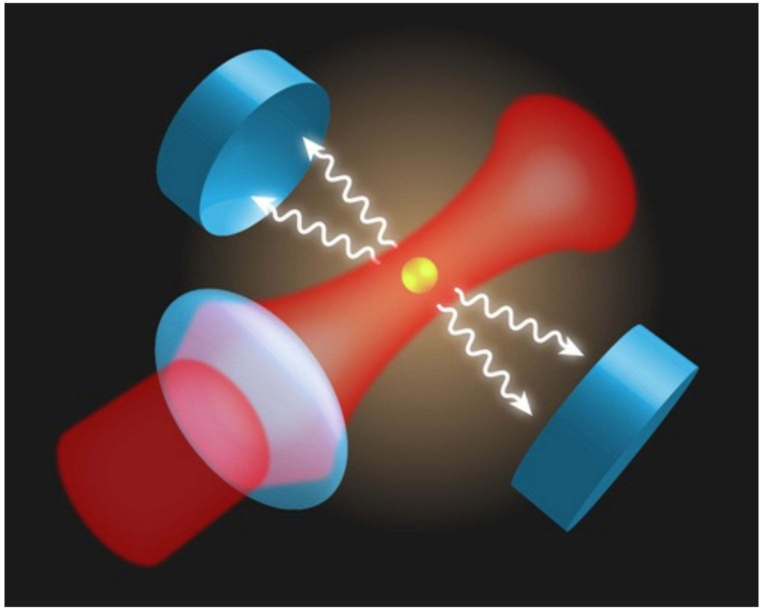
Levitation



Levitated nanosphere
(R. Diehl/R. Reimann/ETH Zurich)

Gonzalez-Ballestero, Aspelmeyer, Novotny, Quidant & Romero-Isart, Science 374, eabg3027 (2021)

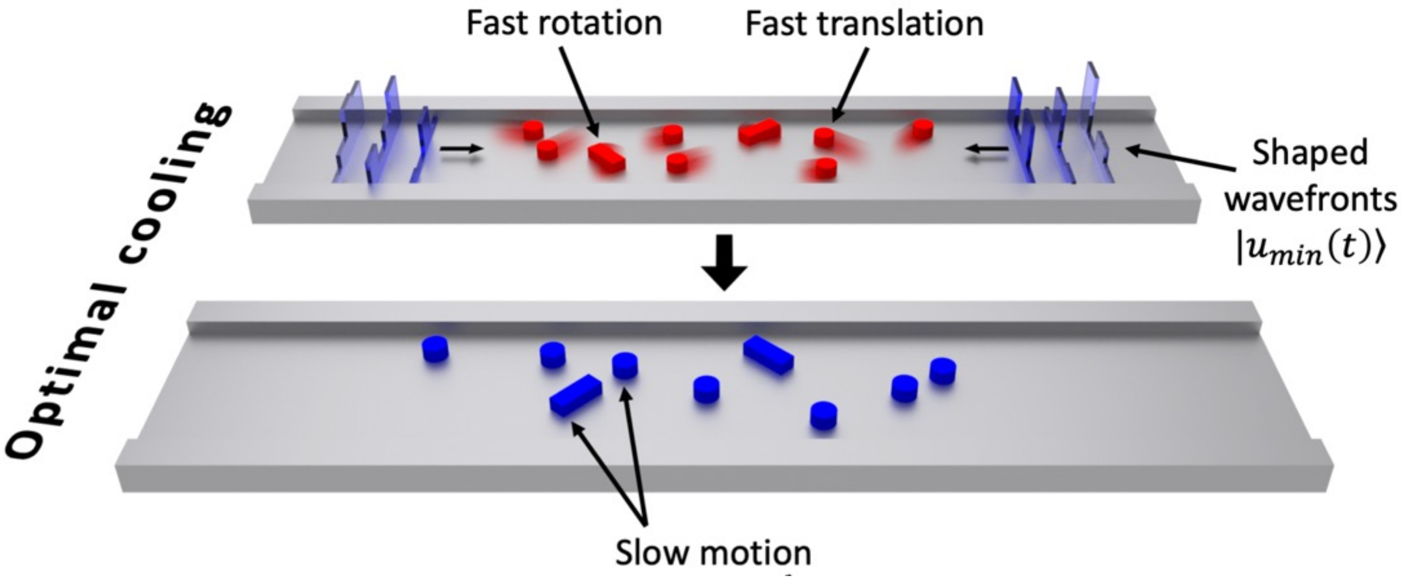
Delić, Reisenbauer, Grass, Kiesel, Vuletić & Aspelmeyer, PRL 122, 123602 (2019).



Optimal cooling

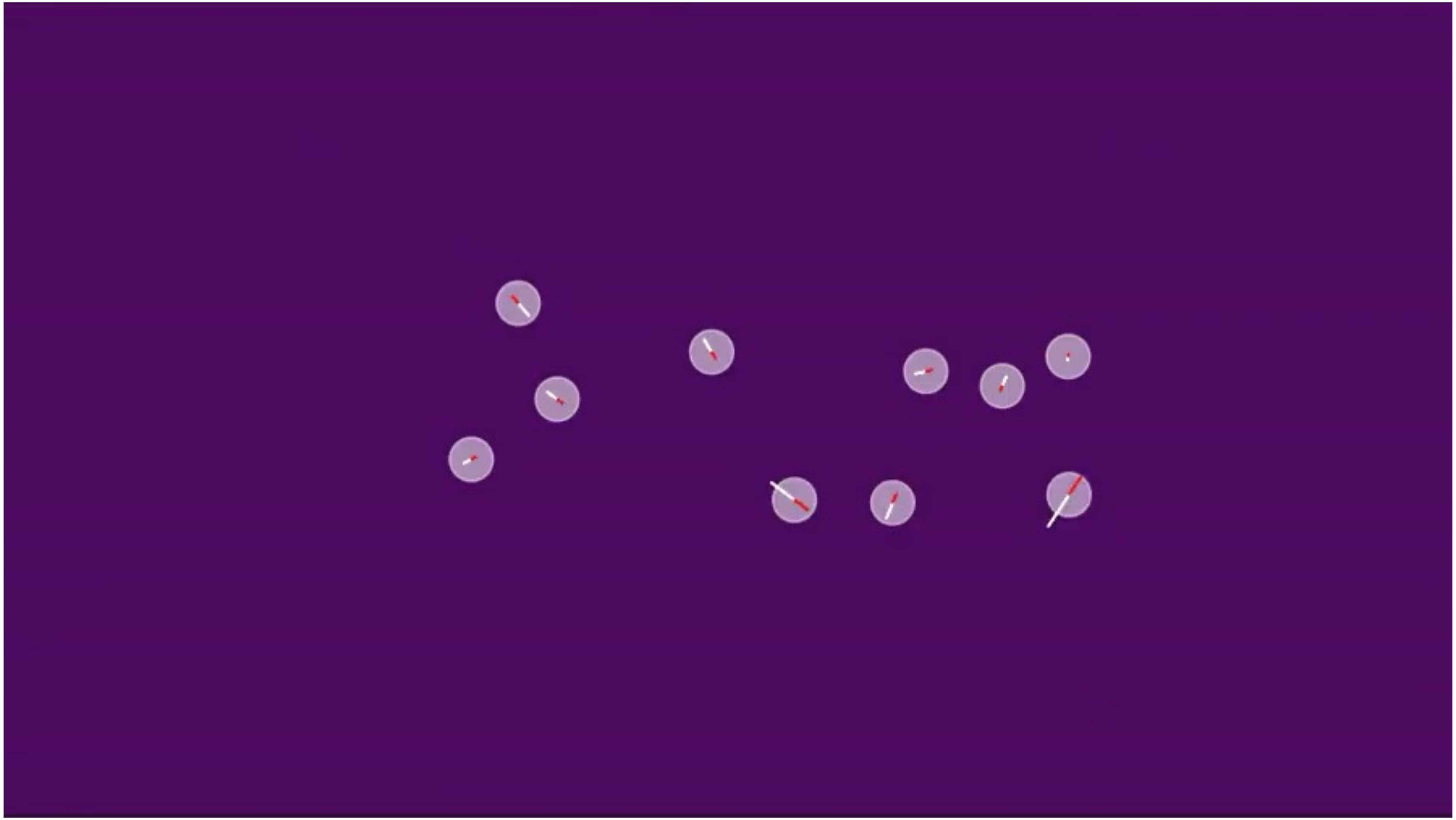
Wigner-Smith "energy shift" operator

$$Q = -iS^{-1}(t)\frac{dS(t)}{dt} \propto \frac{d\,E_{kin}}{dt}$$

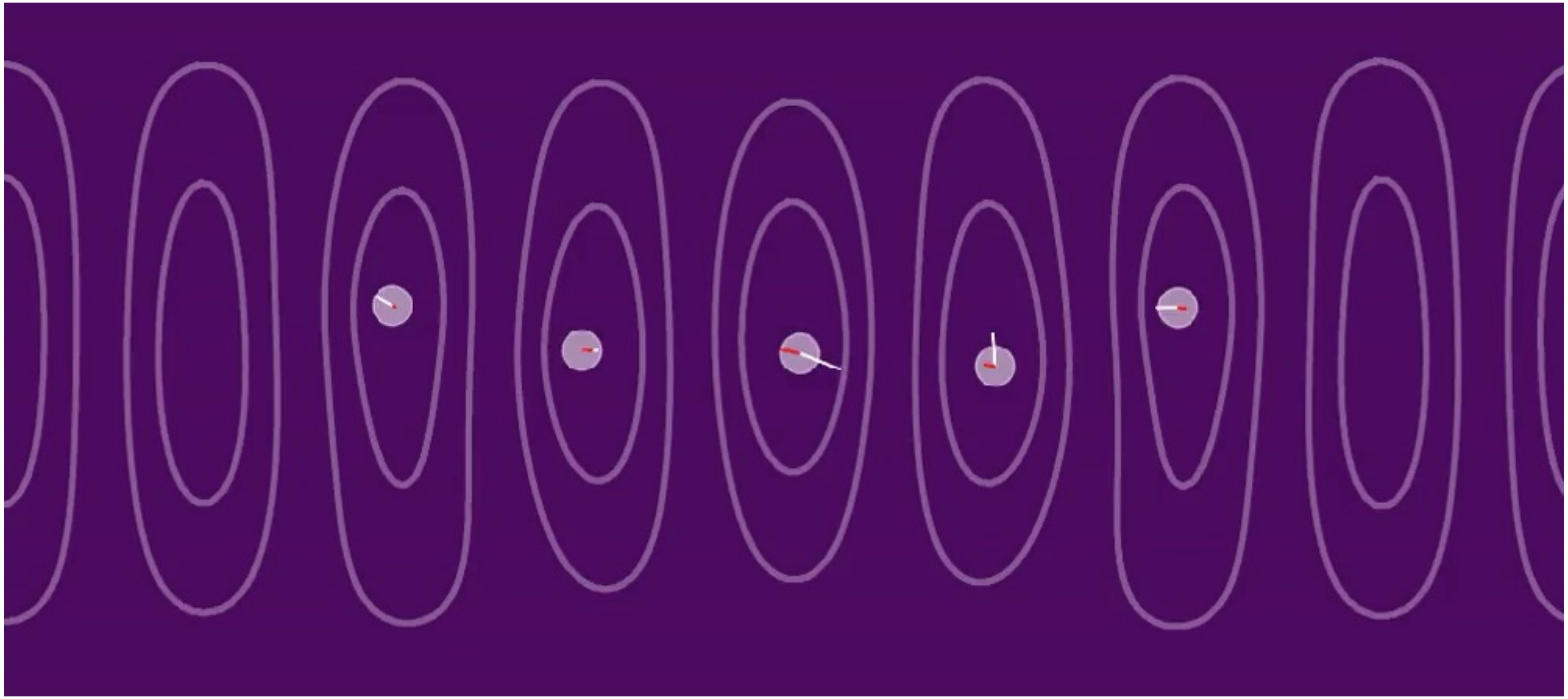


Physics

Hüpfl, Bachelard, Kaczvinszki, Horodynski,
Kühmayer & Rotter, PRL 130, 083203 & PRA 107, 023112 (2023)



Hüpfl, Bachelard, Kaczvinszki, Horodynski,
Kühmayer & Rotter, PRL 130, 083203 & PRA 107, 023112 (2023)

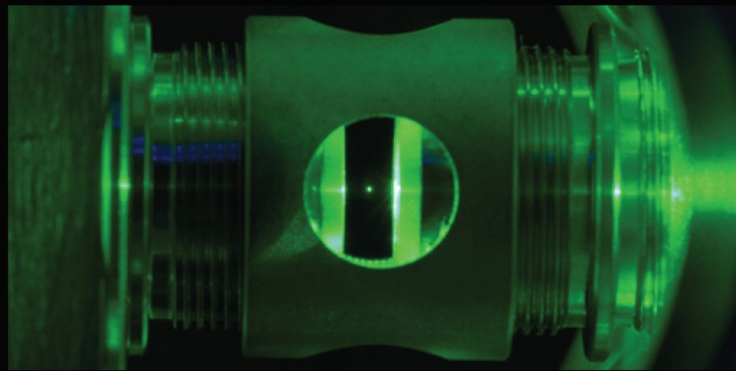


Hüpfl, Bachelard, Kaczvinszki, Horodynski,
Kühmayer & Rotter, PRL 130, 083203 & PRA 107, 023112 (2023)

The Matrix(1999)

Precision measurements

LIGO Livingston (Caltech/MIT/LIGO Lab)



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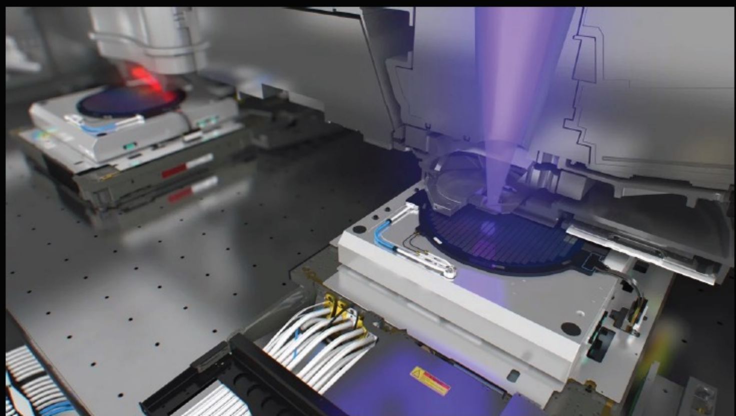
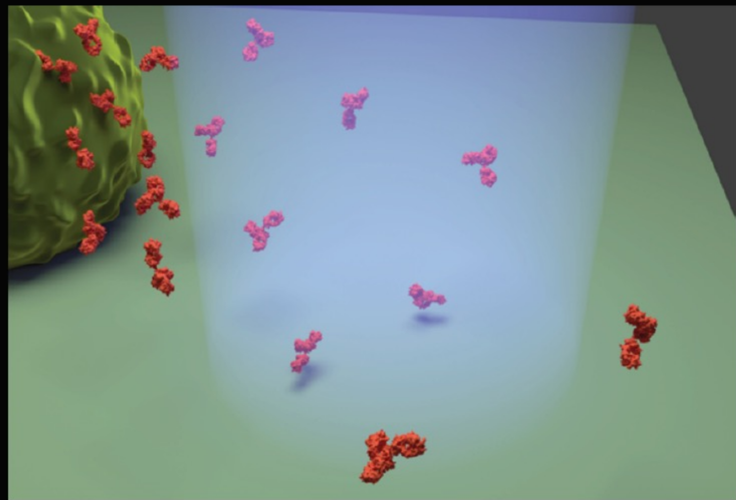
(R. Diehl/R. Reimann/ETH Zurich)

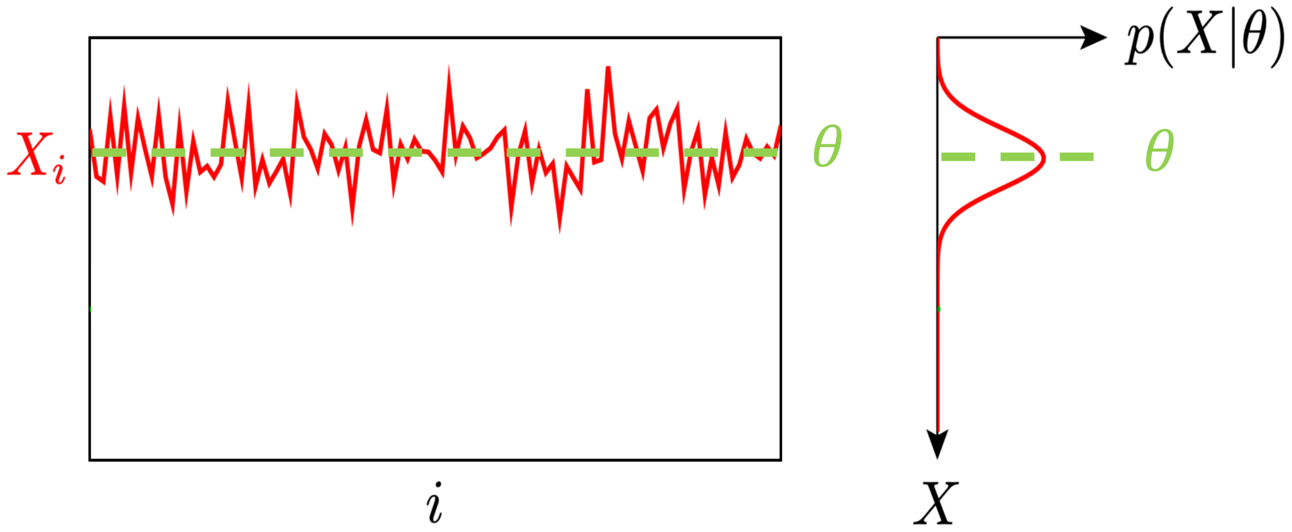


Lithography machine (ASML)



Cellular secretion of proteins
(Nano Lett. 18, 513–519, 2018)

Fisher information

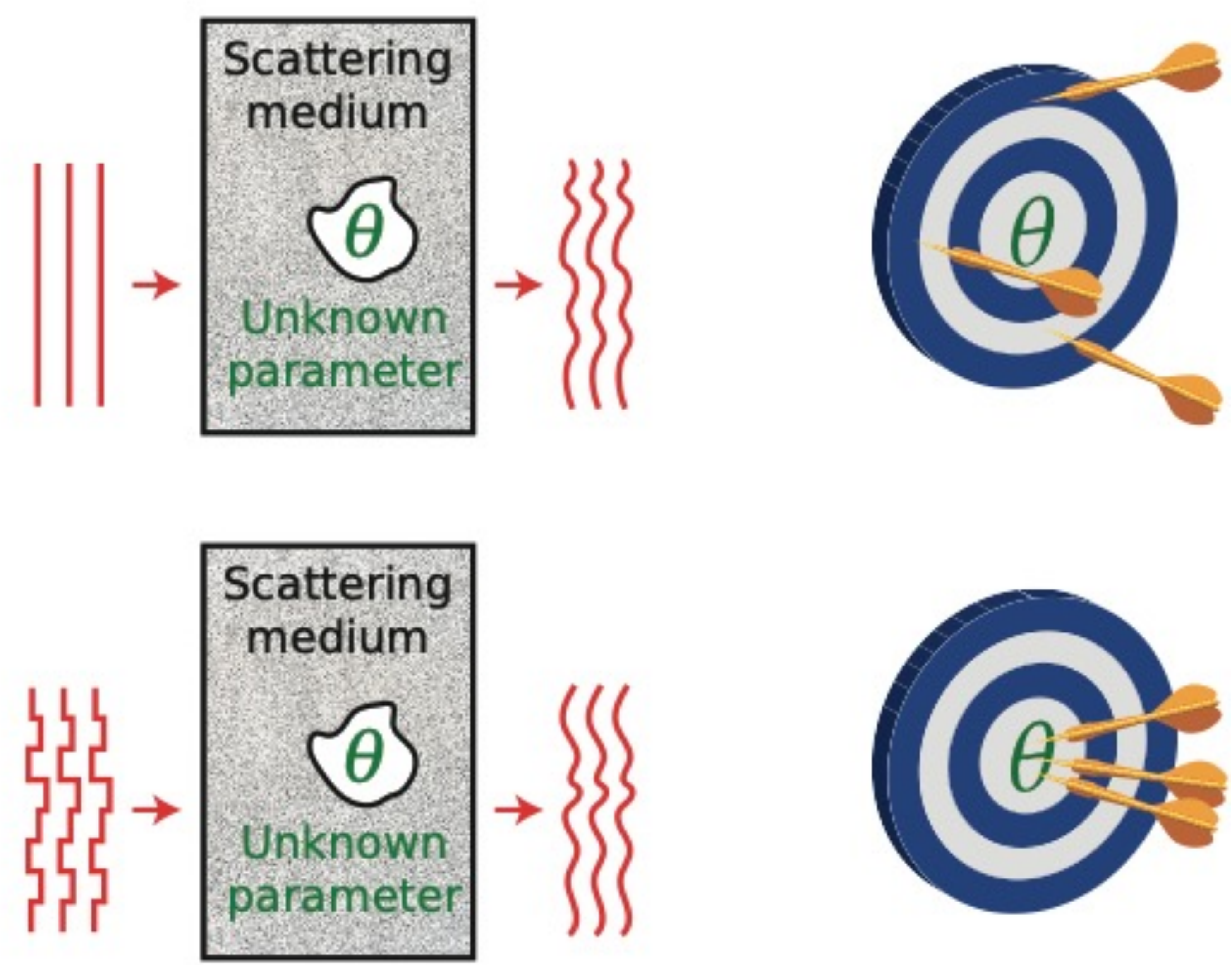


$$J(\theta) = \langle [\partial_\theta \ln p(X;\theta)]^2 \rangle$$ Fisher Information

Cramér-Rao Lower Bound

$$\sigma_{CRB}(\theta) \geq \frac{1}{\sqrt{J(\theta)}}$$

Parameter estimation



Bouchet, Carminati, Mosk, PRL 124, 133903 (2020).

Fisher information

$$\mathcal{J}(\theta) = \mathbb{E}\left(\left[\partial_\theta \ln p(X;\theta)\right]^2\right)$$ Fisher information

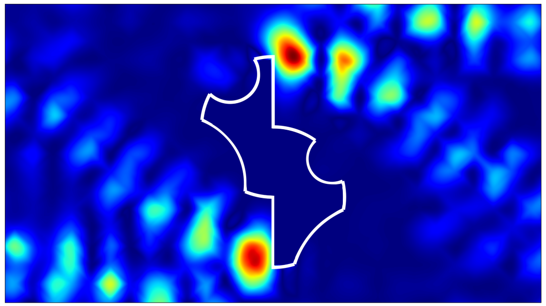
$$E^{\text{out}} = S E^{\text{in}} \qquad \mathcal{J}(\theta) = \langle E^{\text{in}} | F_\theta | E^{\text{in}} \rangle$$

Fisher information operator

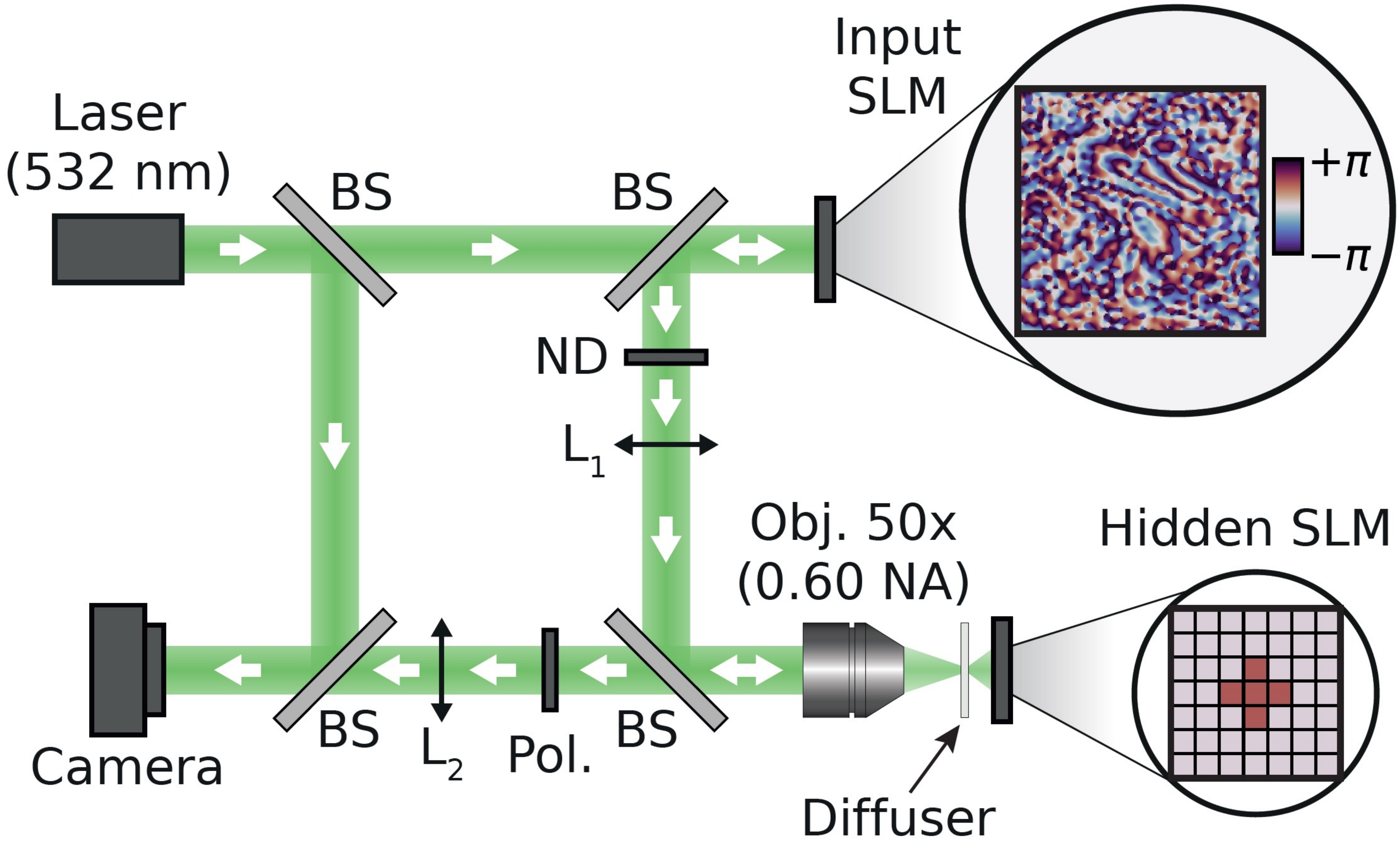
$$F_\theta = (\partial_\theta S)^\dagger \partial_\theta S$$

$$F_\theta = 4\, Q_\theta^2 \qquad Q_\theta = -i S^{-1} \partial_\theta S$$

Maximum information state

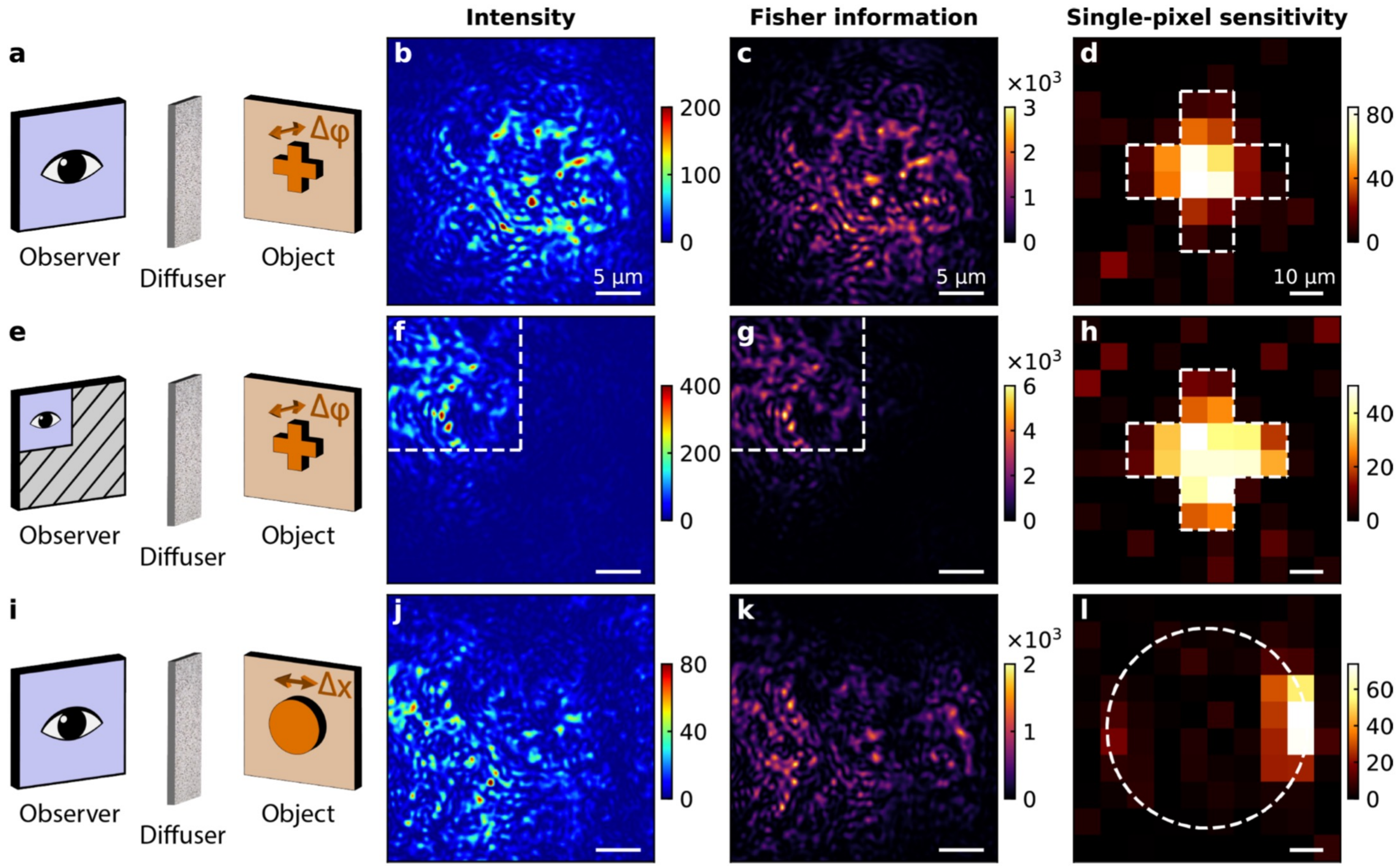


Experiment



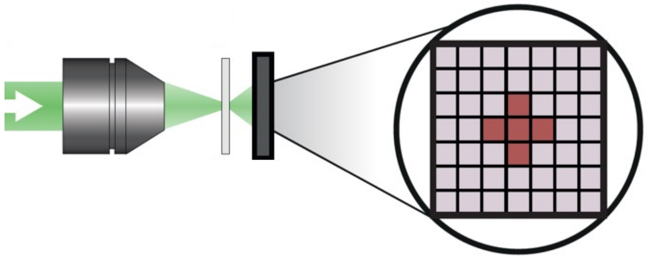
Laser
(532 nm)
BS
BS
Input
SLM
+π
−π
ND
L1
Obj. 50x
(0.60 NA)
Hidden SLM
Camera
BS
L2
Pol.
BS
Diffuser

Experimental data



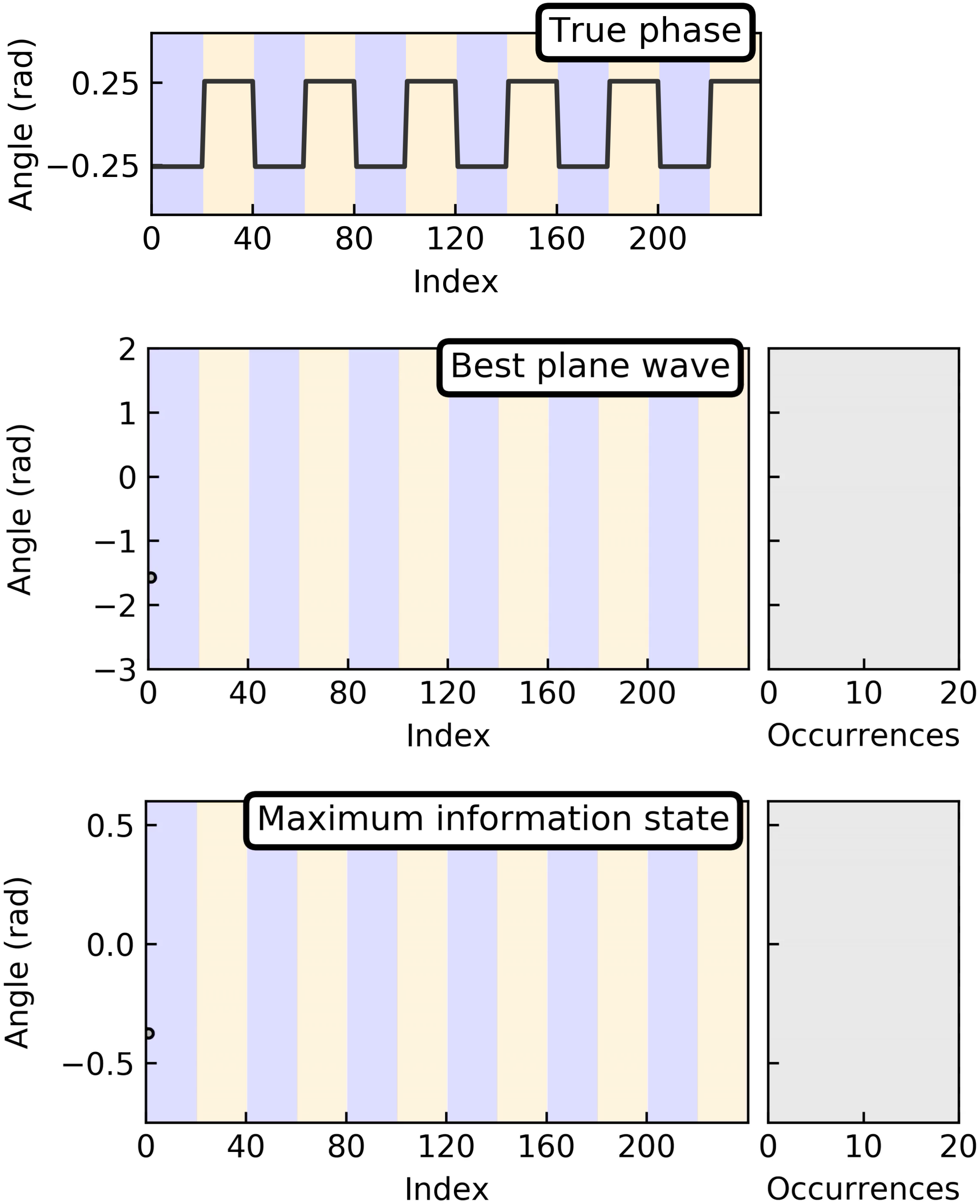
Bouchet, Rotter, Mosk, Nature Physics 17, 564 (2021).

Few photon measurement

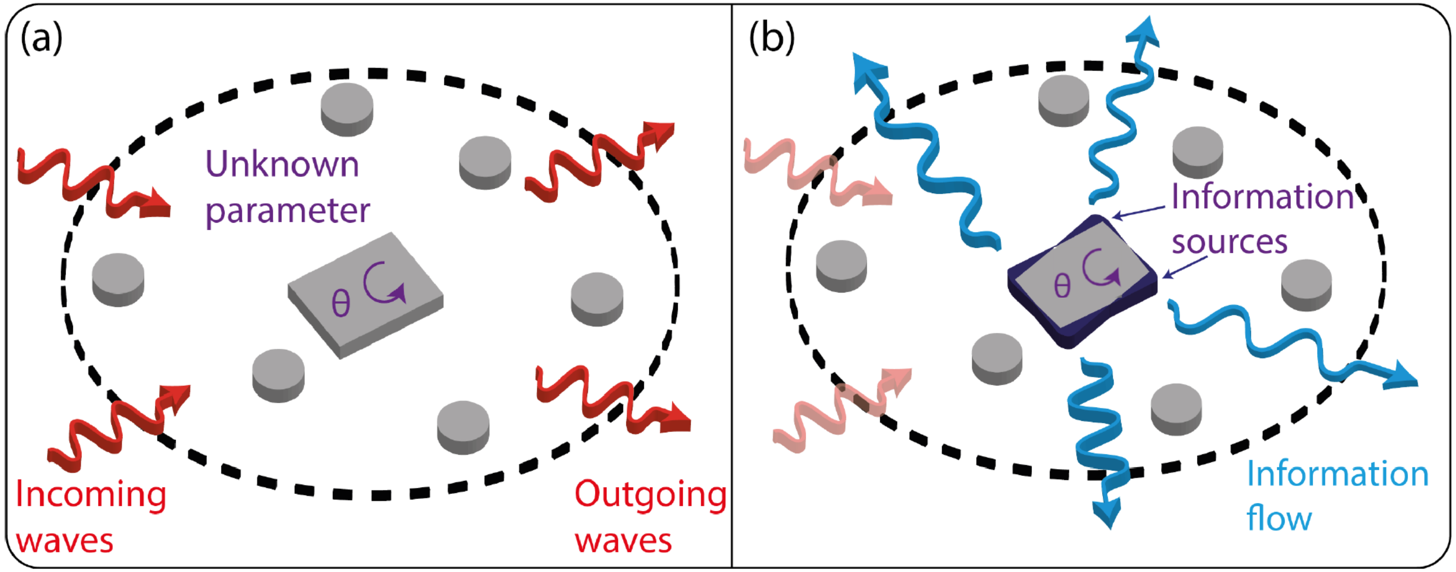


Neutral density filter T = 8.3×10^{-7}

Bouchet, Rotter, Mosk,
Nature Physics 17, 564 (2021)



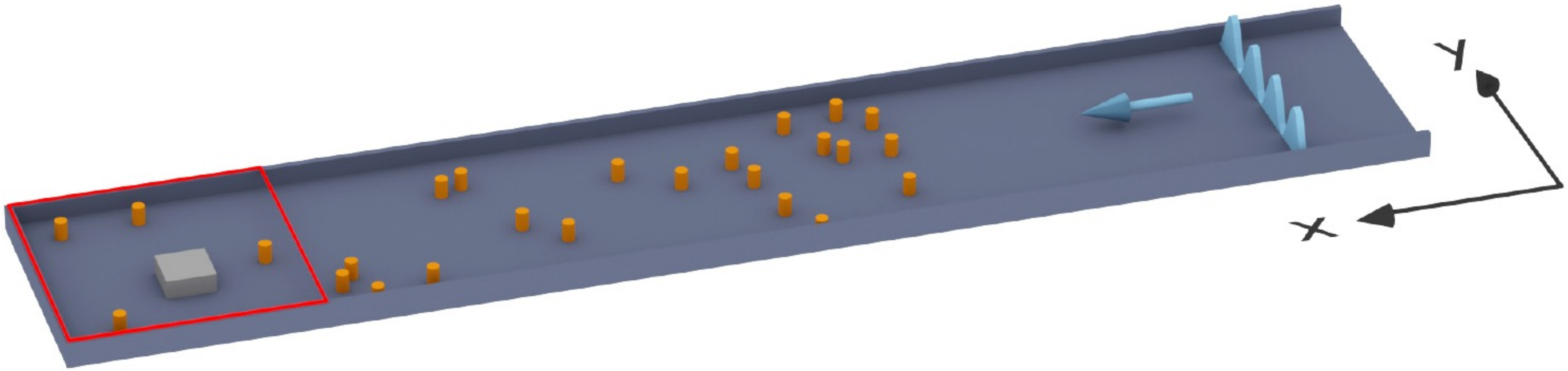
Continuity equation for information flow

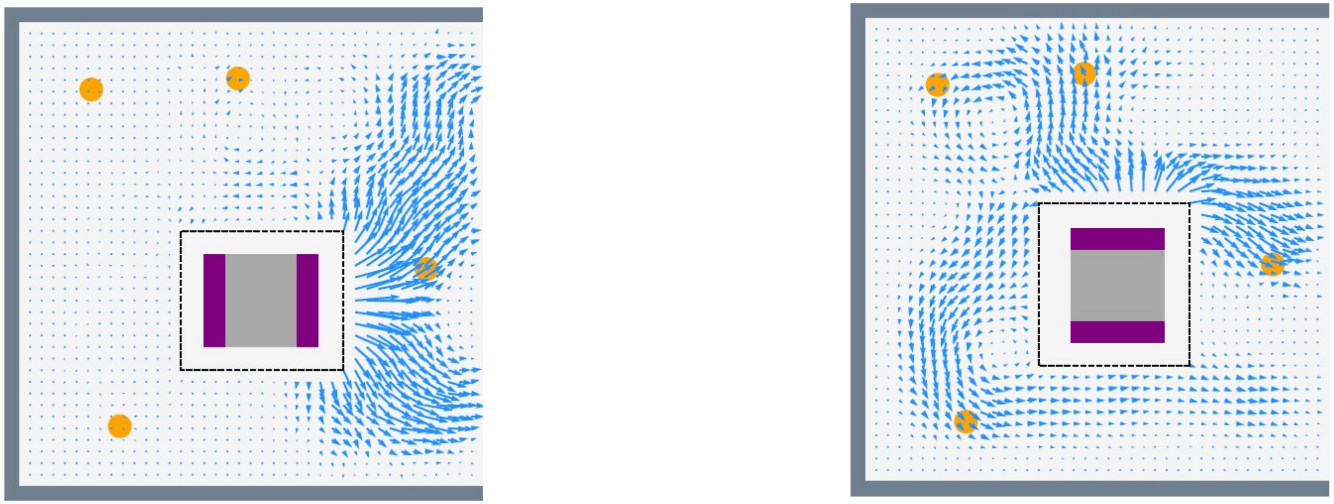


Energy/Power	Fisher-Information
$P = \langle E^{in} \| S^{\dagger} S \| E^{in} \rangle$	$F_{\theta} = 4\langle E^{in} \| (\partial_{\theta} S)^{\dagger} (\partial_{\theta} S) \| E^{in} \rangle / \hbar\omega$
$\boldsymbol{S}^{P} = \mathrm{Re}\,(\boldsymbol{E}_{\omega}^{*} \times \boldsymbol{H}_{\omega})/2$	$\boldsymbol{S}^{FI} = 4\mathrm{Re}\,(\partial_{\theta}\boldsymbol{E}_{\omega}^{*} \times \partial_{\theta}\boldsymbol{H}_{\omega})/(2\hbar\omega)$
$\boldsymbol{\nabla}\boldsymbol{S}^{P} + \partial_t u = \sigma$	$\boldsymbol{\nabla}\boldsymbol{S}^{FI} + \partial_t u^{FI} = \sigma^{FI}$

$$\sigma^{\mathrm{FI}} = 2\hbar^{-1}\mathbf{Im}[(\partial_{\theta}\epsilon)\mathbf{E}_{\omega}^{*} \cdot \partial_{\theta}\mathbf{E}_{\omega}]$$

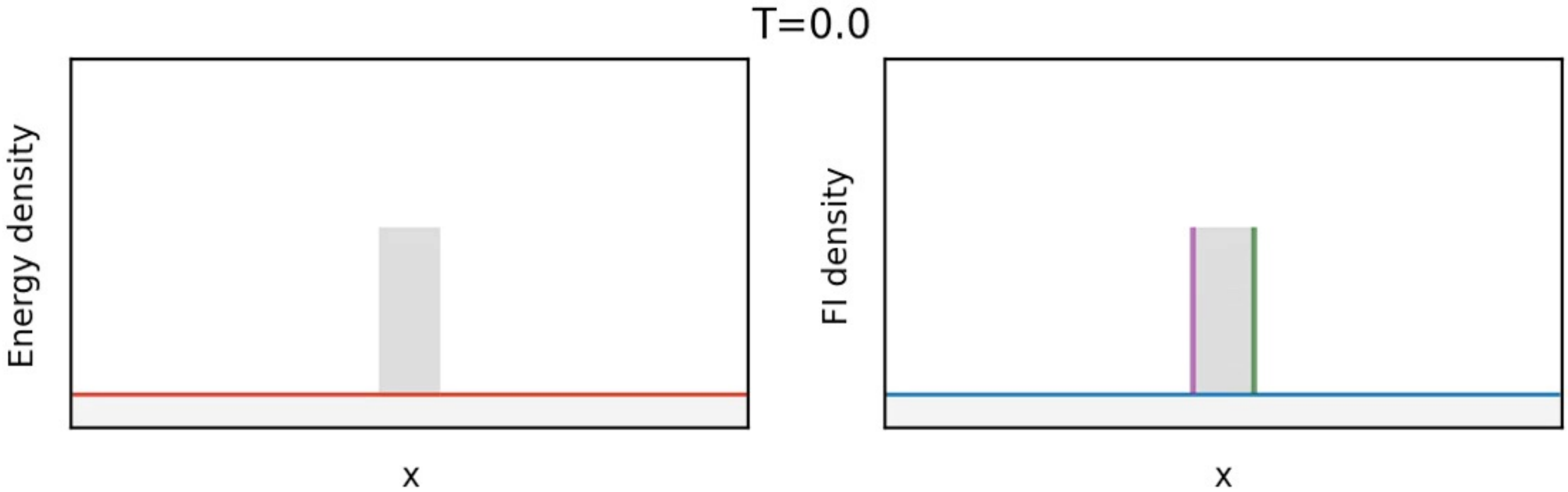
Spatial flow of Fisher information





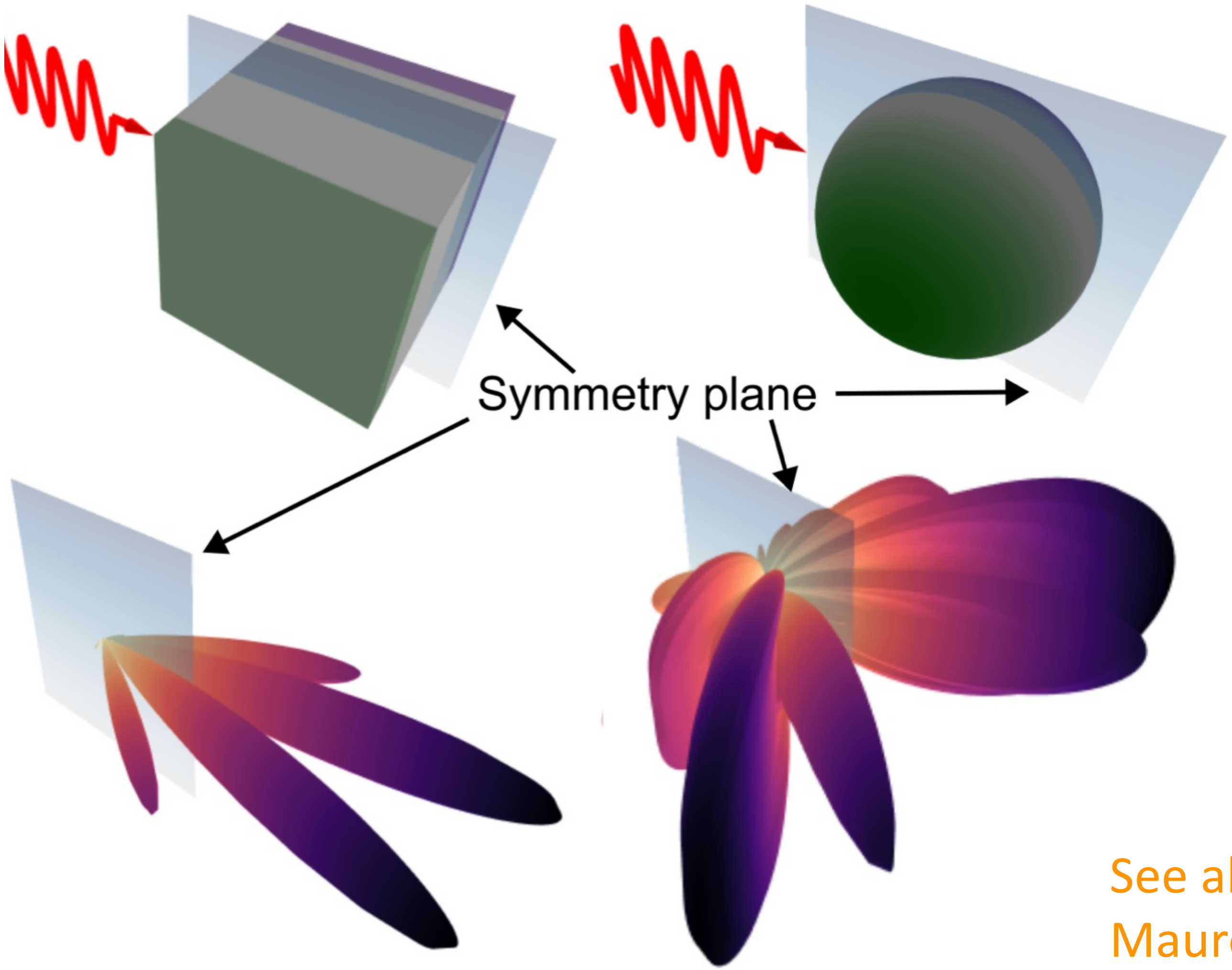
Hüpfl, Russo, Rachbauer, Bouchet, Lu, Kuhl & Rotter
Nature Physics 20, 1294 (2024)

Energy vs. information



Hüpfl, Russo, Rachbauer, Bouchet, Lu, Kuhl & Rotter
Nature Physics 20, 1294 (2024)

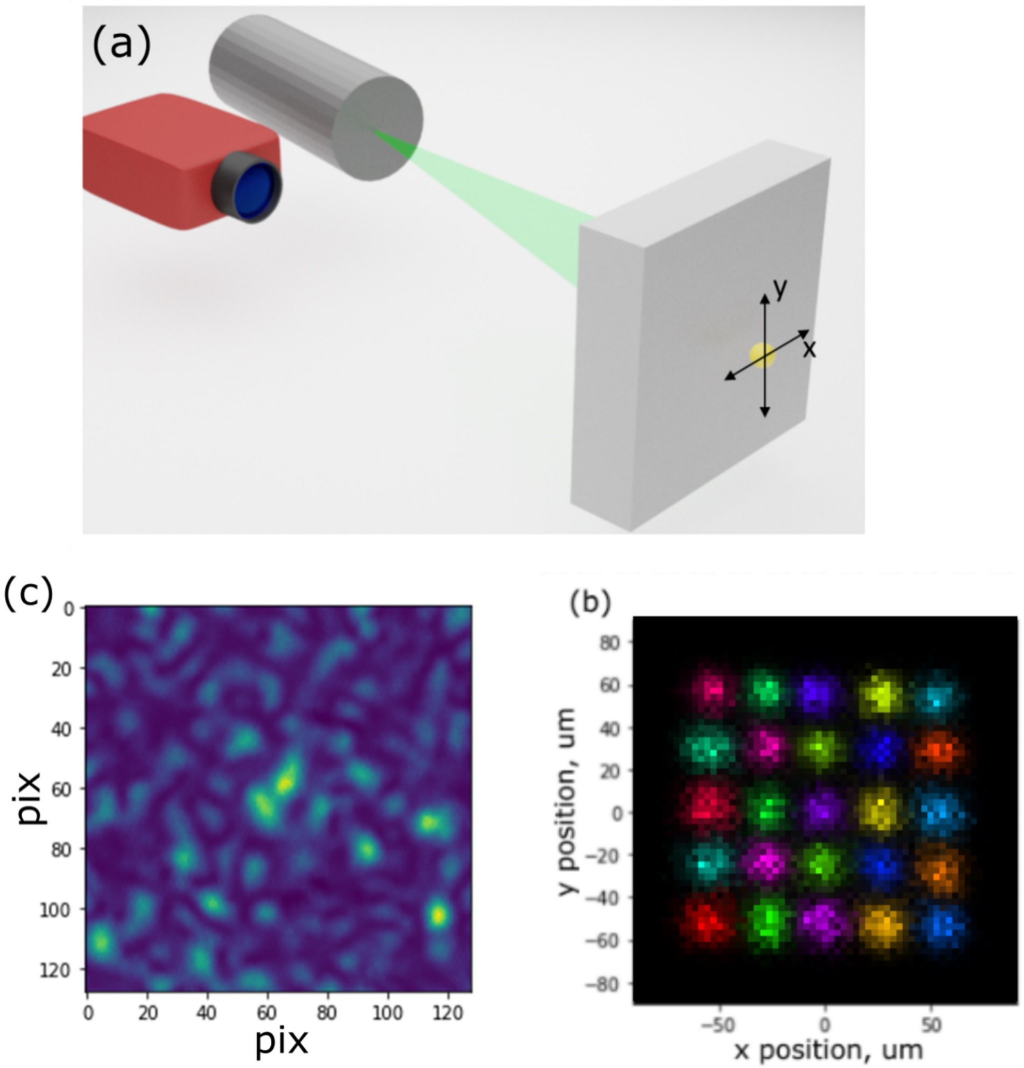
Information radiation patterns

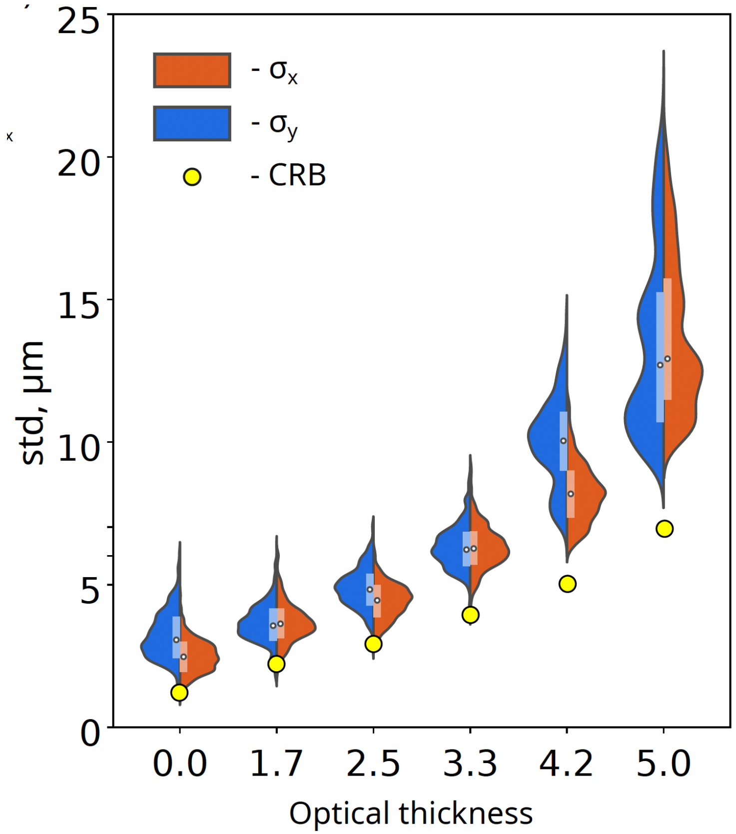


See also:
Maurer, Gonzalez-Ballestero & Romero-Isart,
PRA 108, 033714 (2024)

Ultimate limit of particle localization

Goal: estimate the position of an object behind a dynamic scattering layer





Starshynov, Weimar, Rachbauer, Hackl, Faccio, Rotter & Bouchet, Nature Photonics 19, 593 (2025)

Summary

- Wigner-Smith time-delay operator
- Toolbox for particle manipulation
- Optimal cooling of particle ensemble
- Quantum light fields: Casimir-Polder forces
- Fisher information

REVIEW ARTICLE | INSIGHT
https://doi.org/10.1038/s41567-022-01677-x
nature physics

Shaping the propagation of light in complex media

Hui Cao[1] ✉, Allard Pieter Mosk[2] and Stefan Rotter[3]

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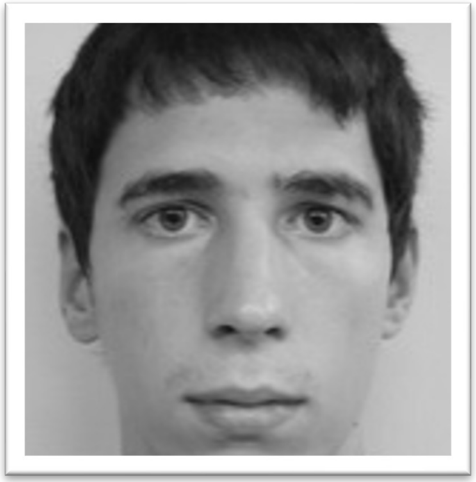

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People involved – Principal modes

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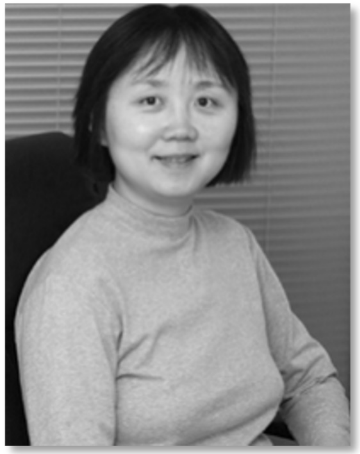

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Thank you for your attention!

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